Naohiro Osamura (Nagoya)

# Novel Loop-diagrammatic approach to QCD $\theta$ parameter and application to the left-right model

**arXiv[2301.13405]** in collaboration with
**Junji Hisano, Teppei Kitahara**
and **Atsuyuki Yamada**

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# Contents

# Strong CP problem

**the theta term:** $\mathcal{L}_{\rm SM} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu}$ $\left( \tilde{G}^{\hat{a}\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^{\hat{a}}_{\rho\sigma} \right)$

- violates $P$ & $T\,(=CP$ under the $CPT$ theorem)
- a physical parameter: $\bar{\theta} = \underbrace{\theta_G}_{\text{bare}} \underbrace{- \arg\det\left[\mathcal{M}_u \mathcal{M}_d\right]}_{\text{the chiral (ABJ) anomaly}}$

S. L. Adler, Phys. Rev. **177** (1969) 2426-38

J. S. Bell, R. Jackiw, Nuovo. Cim. A **60** (1969) 47-61

neutron Electric Dipole Moment (EDM): $d^{\rm exp.}_{\rm neutron} < 1.8 \times 10^{-26}\ e\ {\rm cm}$

C. Abel, *et al.*, Phys. Rev. Lett. 124 (2020) 081803

## Strong CP problem

$\bar{\theta} < 10^{-10} \quad \ll \quad \delta_{\rm CKM} \simeq 66^\circ = 1.2\,{\rm rad} = O(1)$

**Why so little!?**

the $CP$ violating complex phase in the CKM matrix

# Solutions of the strong CP problem

**promising solutions** to the strong CP problem

## axion

dynamical solution

predict a dark matter candidate

**conflict with the quantum gravity**

R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. **38** (1977)

## the left-right model

parity($P$) symmetric model

spontaneous breaking

$$\mathcal{L}_{\rm SMEFT} \ni \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}}$$ small!

free from the quantum gravity

M. A. B. Beg, H. S. Tsao, Phys. Rev. Lett. **41** (1978) 278

**The parity symmetry forbids $G\tilde{G}$!!**

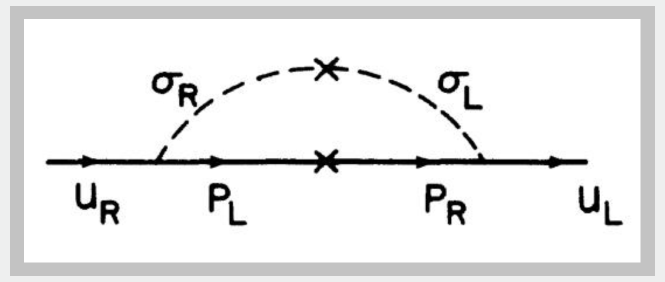
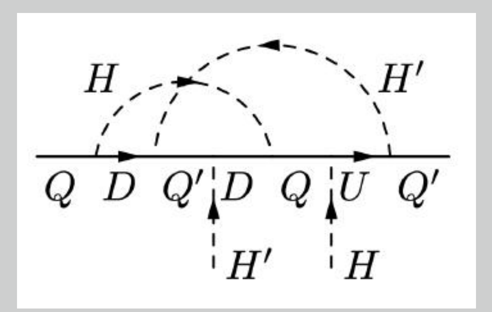
$\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$ : $P$- and $T(CP)$-odd operator

# Left-Right symmetric (LR) model

**LR model :** $SU(3)_C \times SU(2)_L \times \boxed{SU(2)_R} \times U(1)_{B-L}$

$$P_{\rm gen} \;\blacktriangleright\; \mathcal{L}_{\rm LR} \ni \theta_G \cancel{\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}} \tilde{G}^{\hat{a}\mu\nu}$$

L. J. Hall, K. Harigaya, JHEP **10** (2018) 130
N. Craig, *et al.*, JHEP **09** (2021) 130

| a mass matrix in the LR model | | corrections to $\bar{\theta}$ (the chiral anomaly) | |
|---|---|---|---|
| tree (Fujikawa method) | | ✕ | $\because$ $P_{\rm gen}$ ▶ the hermite mass |
| 1-loop corrections |  | ✕ | K. S. Babu, R. N. Mohapatra, Phys. Rev. D **41** (1990) 1286 |
| 2-loop corrections |  | ? | estimated up to a loop function<br>de Vries, et al., arXiv:2109.01630 |

# Our results

- result 1 : We invented the novel method
  to calculate radiative corrections to $\bar{\theta}$.

- result 2 : We applied the novel calculation method
  and estimated the induced $\bar{\theta}$ in the LR model.

# Contents

# Fujikawa method

$T\,(CP)$-odd

$$\mathcal{L}_{\not{P},\not{T}} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - m\bar{\psi}\psi - \boxed{m_{\mathrm{CP}}\bar{\psi}i\gamma_5\psi}$$

mass diagonalize (chiral rotation)

**—Fujikawa method—**

K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

$$\mathcal{L}_{\not{P},\not{T}} \ni \bar{\theta}\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - M\bar{\psi}_M\psi_M \qquad \left(M = m + \frac{m_{\mathrm{CP}}^2}{m}\right)$$

**the physical parameter**

$$\bar{\theta} = \theta_G - \frac{m_{\mathrm{CP}}}{m}$$

# Conventional calculation of $\delta\bar{\theta}$

$T(CP)$-odd

$$\mathcal{L}_{\not{P},\not{T}} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - \left(m + \delta m^{(1)}\right)\bar{\psi}\psi - \boxed{\left(m_{\rm CP} + \delta m^{(1)}_{\rm CP}\right)\bar{\psi}i\gamma_5\psi}$$

radiative corrections

mass diagonalize (chiral rotation)

**—Fujikawa method—**

**+**

**radiative corrections to masses**

$$\mathcal{L}_{\not{P},\not{T}} \ni \bar{\theta}^{\rm loop}\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - M^{(1)}\bar{\psi}_M\psi_M \quad \left(M^{(1)} = (m + \delta m^{(1)}) + \frac{(m_{\rm CP} + \delta m^{(1)}_{\rm CP})^2}{m + \delta m^{(1)}}\right)$$

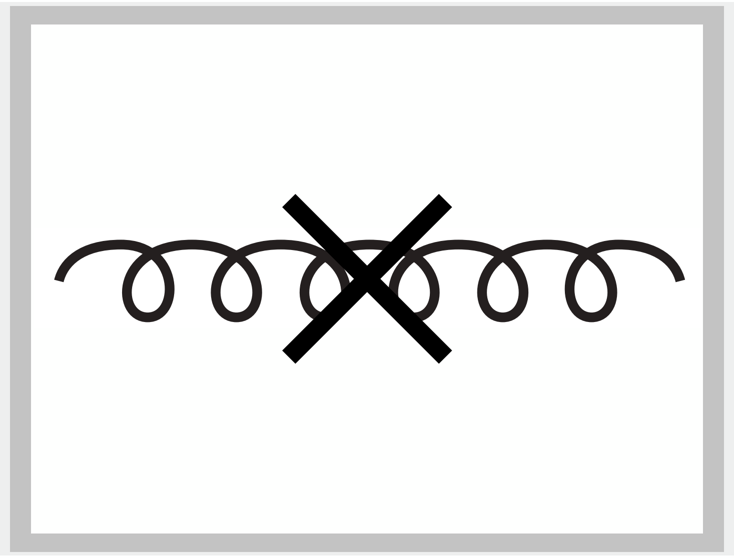
**conventional radiative corrections to $\bar{\theta}$**

$$\bar{\theta}^{\rm loop} \simeq \theta_G - \frac{m_{\rm CP}}{m} - \frac{\delta m^{(1)}_{\rm CP}}{m} + \frac{m_{\rm CP}}{m}\frac{\delta m^{(1)}}{m}$$

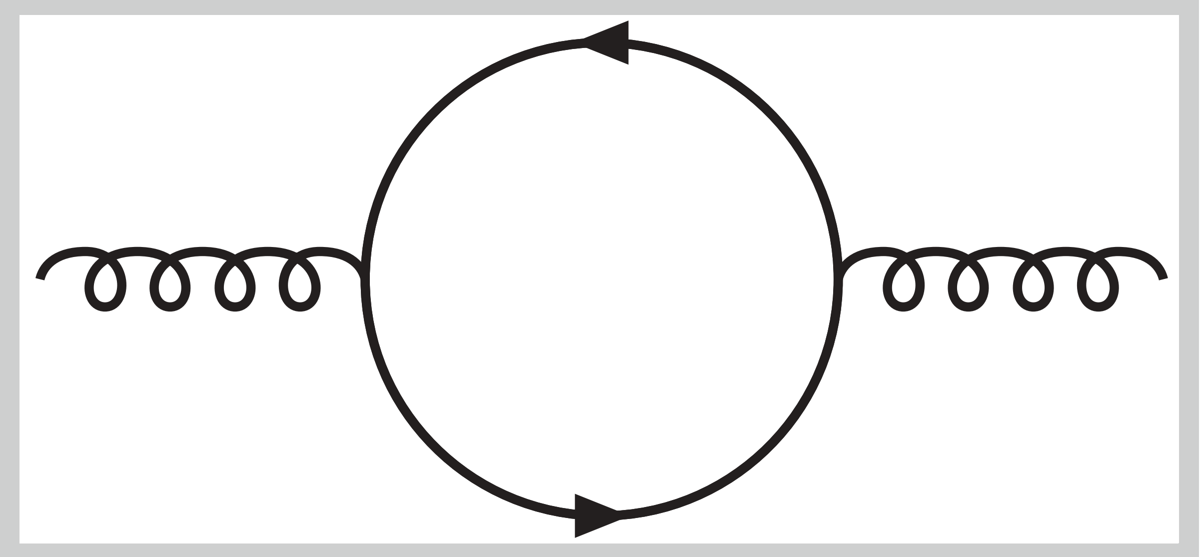
# Diagrammatic approach to $\bar{\theta}$ corrections

$$\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$

$\bar{\theta} = \quad \theta_G$

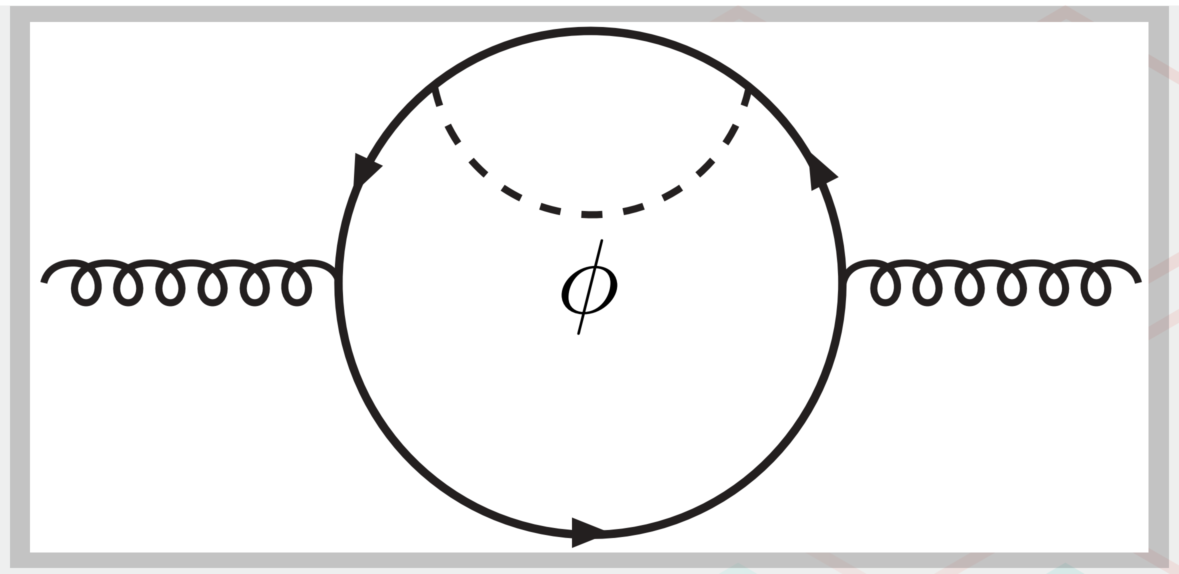


$-\dfrac{m_{\mathrm{CP}}}{m}$



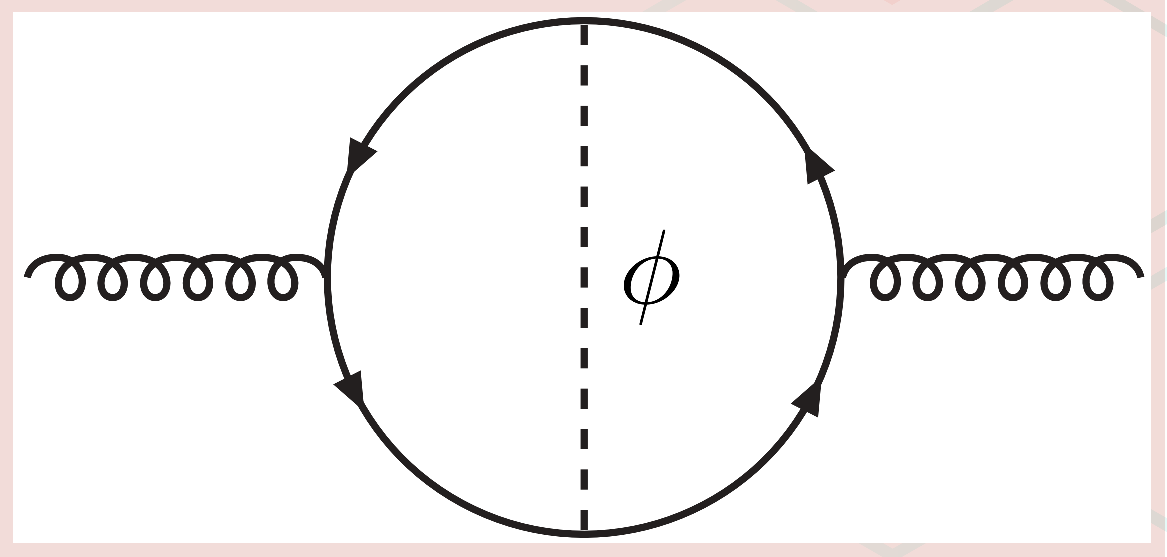


$-\dfrac{\delta m^{(1)}_{\mathrm{CP}}}{m} + \dfrac{m_{\mathrm{CP}}}{m}\dfrac{\delta m^{(1)}}{m}$



(assumption: a scalar $\phi$ which interacts with $\psi$)

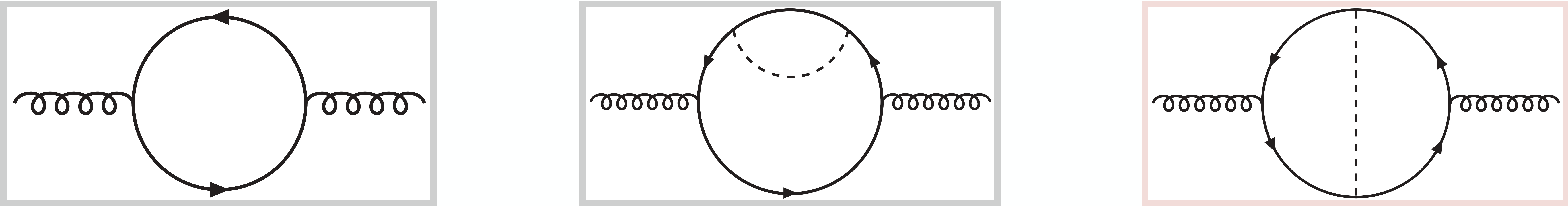
$+$ **New!**





$+\cdots$

# Difficulty in calculating the loop diagrams



**difficulty:** the theta term cannot be derived perturbatively.

$\because$ total derivative $G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} = \partial^\mu \epsilon_{\mu\nu\rho\sigma}\left(A^{\hat{a}}_\nu G^{\hat{a}}_{\rho\sigma} - \frac{g_s}{3} f^{\hat{a}\hat{b}\hat{c}} A^{\hat{a}}_\nu A^{\hat{b}}_\rho A^{\hat{a}}_\sigma\right)$ $\sum p^\mu = 0$

**strategy**

- building a gluon effective theory described by not gauge field $A^{\hat{a}}_\mu$ but the field strength $G^{\hat{a}}_{\mu\nu}$
- temporarily breaking the translation symmetry

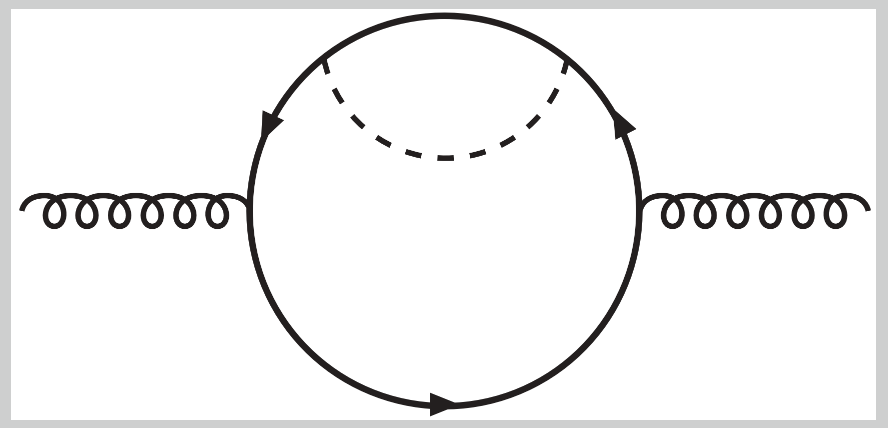
# Fock-Schwinger gauge method

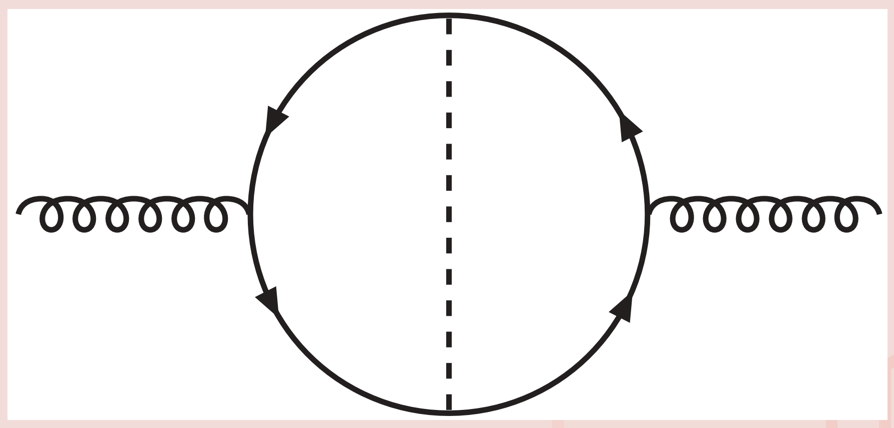
**Fock-Schwinger gauge:** $(x^\mu - x_0^\mu) A_\mu^{\hat{a}}(x) = 0$ V. A. Novikov, et al., Fortsch. Phys. **32** (1984) 585

- gauge fixing $A_\mu^{\hat{a}}(x) = \frac{1}{2}(x^\nu - x_0^\nu) G_{\nu\mu}^{\hat{a}}(x_0) + \cdots$
- breaks the translation symmetry, but it revives in the result of gauge invariant quantities

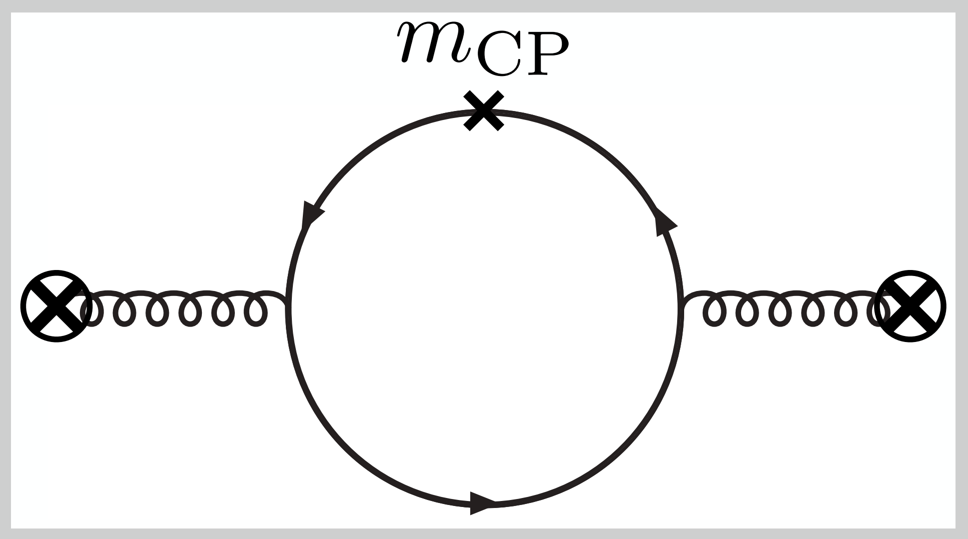
S. N. Nikolaev, et al., Nucl. Phys. **B** 213 (1983) 285-304

- calculable





$$\mathcal{L} = \bar{\psi} i \not{D} \psi - m \bar{\psi}\psi - m_{\mathrm{CP}} \bar{\psi} i \gamma_5 \psi - \frac{1}{4} G_{\mu\nu}^{\hat{a}} G^{\hat{a}\mu\nu} + \theta_G \frac{\alpha_s}{8\pi} G_{\mu\nu}^{\hat{a}} \tilde{G}^{\hat{a}\mu\nu}$$

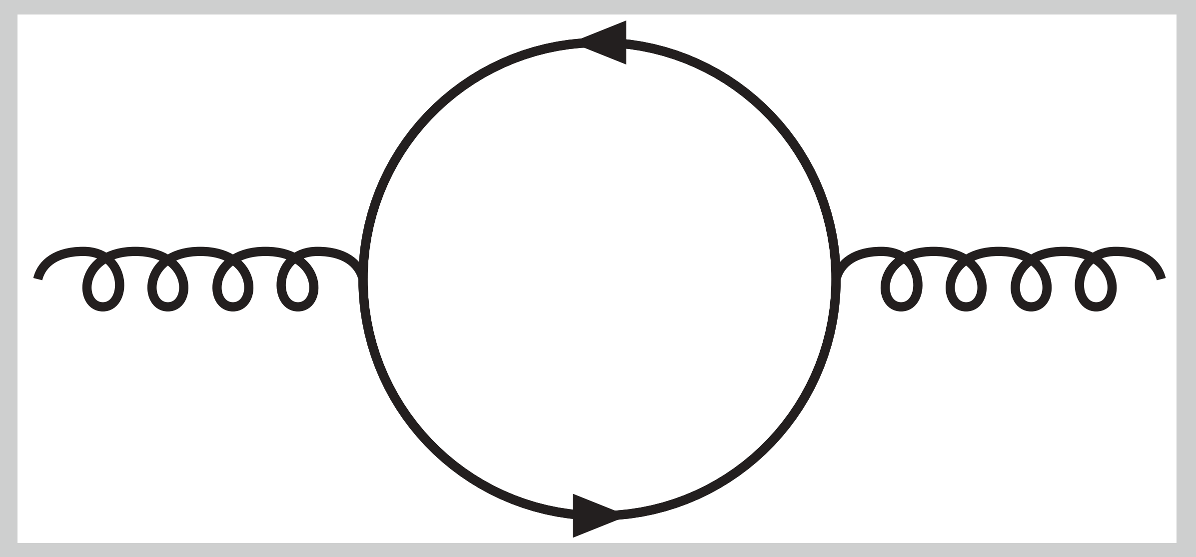
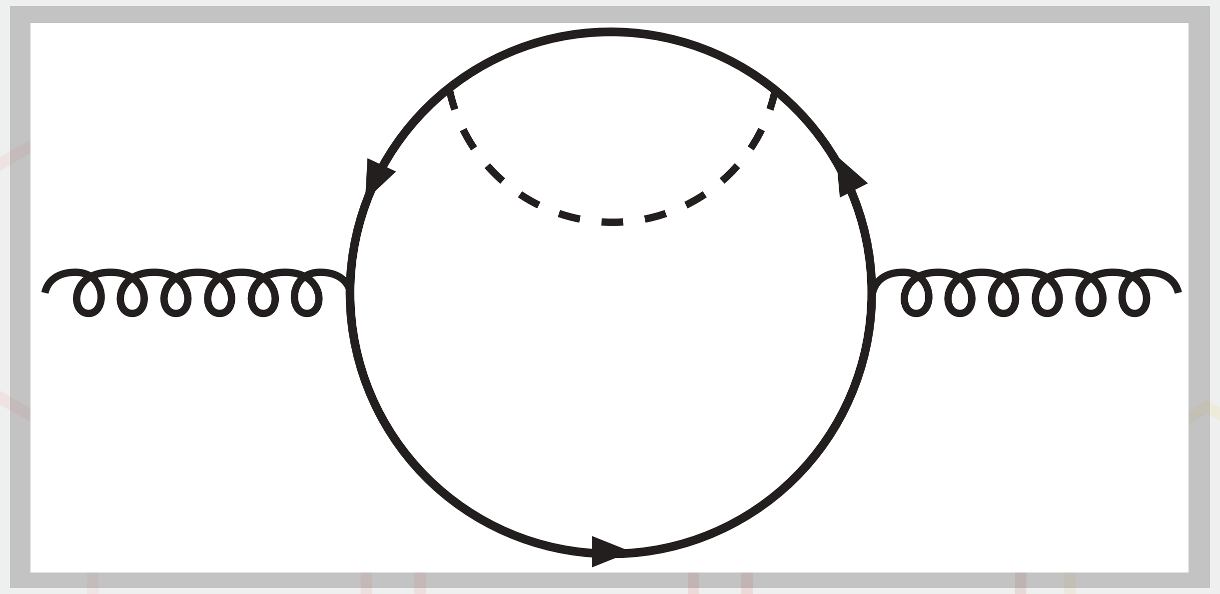
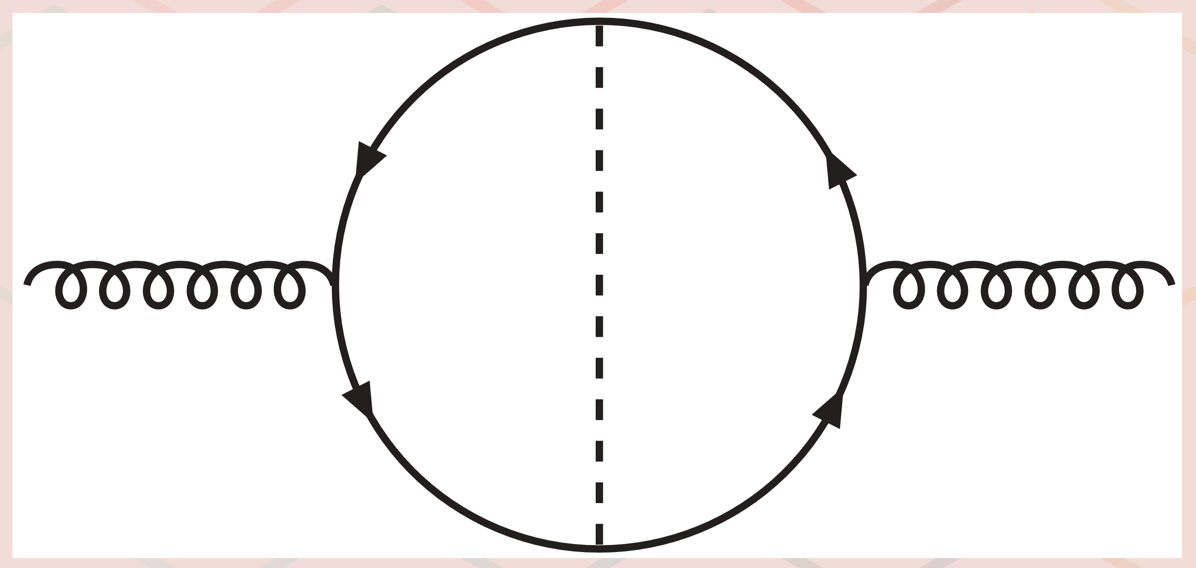



$$-\frac{m_{\mathrm{CP}}}{m}$$

(Fujikawa method)

**consistent!**

# 1st result

| a mass (matrix) | corrections to $\bar{\theta}$ (the chiral anomaly) | | **1. the diagrammatic method (Fock-Schwinger gauge)** |
|---|---|---|---|
| tree $(m,\, m_{\mathrm{CP}})$ | Fujikawa method | consistent! |  |
| 1-loop corrections $(\delta m,\, \delta m_{\mathrm{CP}})$ | Fujikawa method $+$ loop masses | |  |
| ✕ | ✕ | |  **New!!** |

# Contents

# Minimal LR model

SM: $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

**LR model:** $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory

$P_{\text{gen}}$ **: generalized parity symmetry**

$SU(2)_L$ singlet $\longrightarrow$ $SU(2)_R$ doublet

LR model $\xrightarrow[\langle H' \rangle = (0, v')]{}$ SM (low energy EFT)

$SU(2)_{L/R}$ singlet, vector-like(VL) quarks

(SM quark masses by seesaw mechanism)

| | $SU(3)_C$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | $U(1)_Y$ |
|---|---|---|---|---|---|
| $Q_L^i \equiv (u_L^i, d_L^i)^T$ | $\Box$ | $\Box$ | $\mathbf{1}$ | $1/6$ | $(1/6, 1/6)$ |
| $Q_R^i \equiv (u_R^i, d_R^i)^T$ | $\Box$ | $\mathbf{1}$ | $\Box$ | $1/6$ | $(2/3, -1/3)$ |
| $H$ | $\mathbf{1}$ | $\Box$ | $\mathbf{1}$ | $1/2$ | $(1/2, 1/2)$ |
| $H'$ | $\mathbf{1}$ | $\mathbf{1}$ | $\Box$ | $1/2$ | $(1, 0)$ |
| $U_L^a$ | $\Box$ | $\mathbf{1}$ | $\mathbf{1}$ | $2/3$ | $2/3$ |
| $U_R^a$ | $\Box$ | $\mathbf{1}$ | $\mathbf{1}$ | $2/3$ | $2/3$ |
| $D_L^a$ | $\Box$ | $\mathbf{1}$ | $\mathbf{1}$ | $-1/3$ | $-1/3$ |
| $D_R^a$ | $\Box$ | $\mathbf{1}$ | $\mathbf{1}$ | $-1/3$ | $-1/3$ |

$\gtrsim 1\text{TeV}$

# Seesaw mechanism

mass matrix (light flavor: $Q^i_{L/R}$, heavy flavor: $U^a_{L/R}$, $D^a_{L/R}$)

$$
\begin{aligned}
-\mathcal{L}_Y = & \bar{Q}^i_L x^{ia}_u U^a_R \tilde{H} + \bar{Q}^i_R x^{ia}_u U^a_L \tilde{H}' + M^a_u \bar{U}^a_L U^a_R \\
& + \bar{Q}^i_L x^{ia}_d D^a_R H + \bar{Q}^i_R x^{ia}_d D^a_L H' + M^a_d \bar{D}^a_L D^a_R \\
& + \mathrm{h.c.}
\end{aligned}
$$

Yukawa (light × heavy × Higgs) Dirac masses

**seesaw mechanism**

$$
\left(\overline{u}^i_L, \overline{U}^a_L\right) \begin{pmatrix} 0 & x^{ib}_u v \\ x^{\dagger aj}_u v' & M^a_u \delta^{ab} \end{pmatrix} \begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix}
$$

$$
\left(\overline{d}^i_L, \overline{D}^a_L\right) \begin{pmatrix} 0 & x^{ib}_d v \\ x^{\dagger aj}_d v' & M^a_d \delta^{ab} \end{pmatrix} \begin{pmatrix} d^j_R \\ D^b_R \end{pmatrix}
$$

- up-type VL quark mass hierarchy ⟶ SM up-type quark mass hierarchy

$$M^1_u \gg M^2_u \gg M^3_u \qquad m_u \ll m_c \ll m_t$$

- down-type Yukawa ($x_d$) components ⟶ SM down-type quark mass hierarchy (∵ mild)

$$M^1_d = M^2_d = M^3_d \qquad m_d < m_s < m_b$$

# Seesaw mechanism

mass matrix (light flavor: $Q^i_{L/R}$, heavy flavor: $U^a_{L/R}$, $D^a_{L/R}$)

$$
\begin{aligned}
-\mathcal{L}_Y = & \bar{Q}^i_L x^{ia}_u U^a_R \tilde{H} + \bar{Q}^i_R x^{ia}_u U^a_L \tilde{H}' + M^a_u \bar{U}^a_L U^a_R \\
& + \bar{Q}^i_L x^{ia}_d D^a_R H + \bar{Q}^i_R x^{ia}_d D^a_L H' + M^a_d \bar{D}^a_L D^a_R \\
& + \mathrm{h.c.}
\end{aligned}
$$

Yukawa (light × heavy × Higgs) Dirac masses

**seesaw mechanism**

$$
\left(\overline{u}^i_L, \overline{U}^a_L\right)\begin{pmatrix} 0 & x^{ib}_u v \\ x^{\dagger aj}_u v' & M^a_u \delta^{ab} \end{pmatrix}\begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix}
$$

$$
\left(\overline{d}^i_L, \overline{D}^a_L\right)\begin{pmatrix} 0 & x^{ib}_d v \\ x^{\dagger aj}_d v' & M^a_d \delta^{ab} \end{pmatrix}\begin{pmatrix} d^j_R \\ D^b_R \end{pmatrix}
$$

## CP phases

$$
x_u = V^\dagger_{\mathrm{CKM}} \frac{\sqrt{m_u}}{\sqrt{v}} \overline{\Phi}(\theta_{u3}, \theta_{u8}) V_U \frac{\sqrt{M_u}}{\sqrt{v'}}, \qquad 1(V_{\mathrm{CKM}}) + 2(\theta_{u3}, \theta_{u8}) + 1(V_U) = 4
$$

$$
x_d = \frac{\sqrt{m_d}}{\sqrt{v}} \cancel{\overline{\Phi}(\theta_{d3}, \theta_{d8}) V_D} \frac{\sqrt{M_d}}{\sqrt{v'}} \qquad \cancel{2(\theta_{d3}, \theta_{d8}) + 1(V_D) = 3}
$$

$(V_{U/D}$: CKM-like matrix, $\overline{\Phi}(\theta_{q3}, \theta_{q8})$: two $U(1)$ phases$)$

**Assumptions:** $\tilde{M} \equiv M^a_d = M^1_u \gg M^2_u \gg M^3_u$

# $\bar{\theta}$ parameter in the minimal LR model

**tree**

$$\mathcal{L}_{\mathrm{Left-Right}} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu}$$

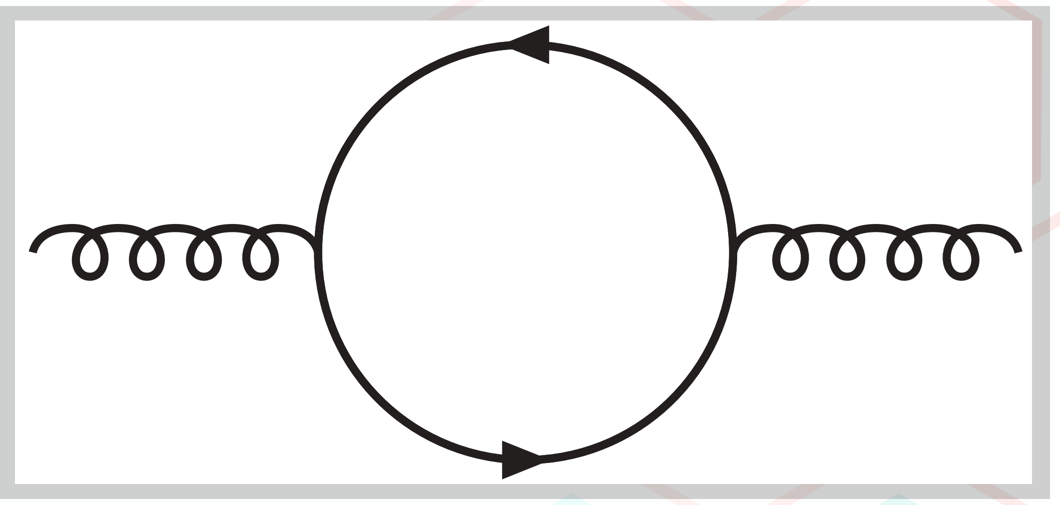
**prohibited by $P_{\mathrm{gen}}$**

**1-loop (Fujikawa method)**

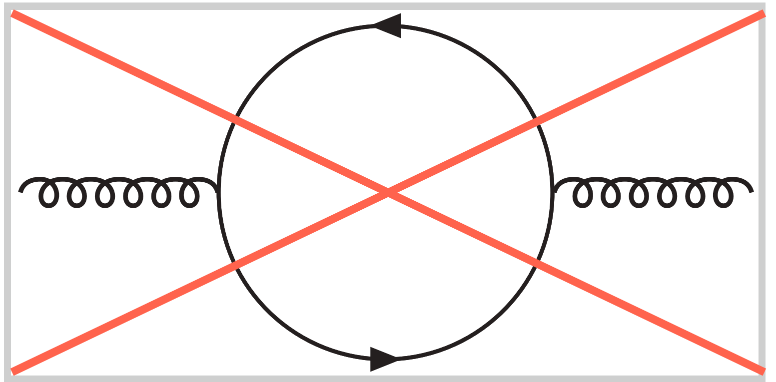
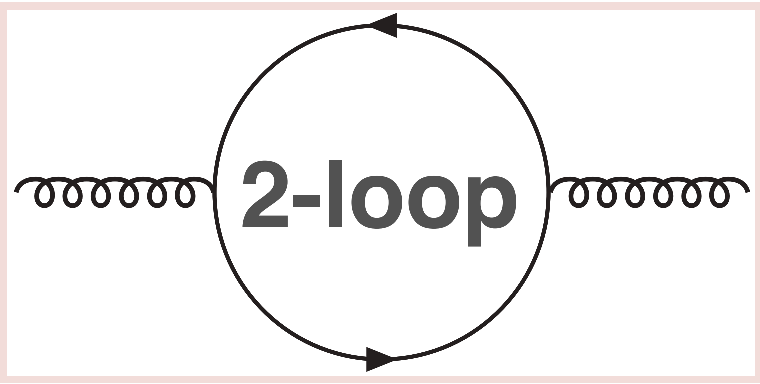
the mass matrix

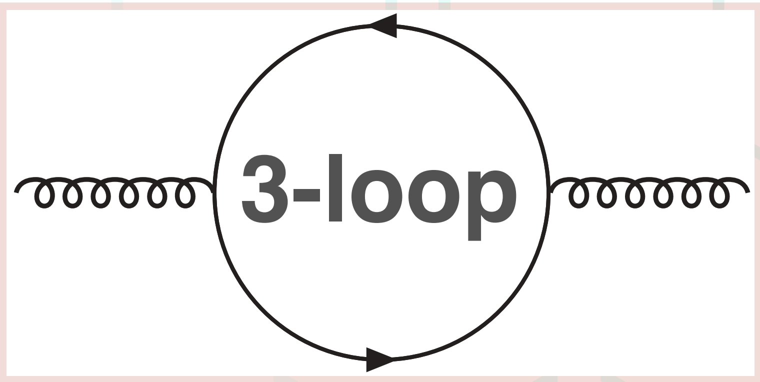
$$\left(\overline{u}^i_L, \overline{U}^a_L\right) \begin{pmatrix} 0 & x_u^{ib} v \\ x_u^{\dagger aj} v' & M_u^a \delta^{ab} \end{pmatrix} \begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix} \equiv \overline{\mathcal{U}}^p_L \mathcal{M}_u^{(0)pq} \mathcal{U}^q_R$$

$$\arg\det\left[\mathcal{M}_u \mathcal{M}_d\right] = 0 \quad \Longleftrightarrow$$



$= 0$

$\bar{\theta} = \cancel{\theta_G} +$  $+$ 

$+$ 

$+ \cdots$

# Higher dimensional operator (HDO) inducing $\theta$

the spontaneous $P_{\text{gen}}$ breaking ▶ $\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$

HDOs inducing $\bar{\theta}$: $\sum_{n=1} C_n \frac{|H'|^{2n} - \cancel{|H|^{2n}}}{M_q^{2n}} \frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} \to \sum_{n=1} C_n \frac{v'^{2n} - \cancel{v^{2n}}}{M_q^{2n}} \frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$ due to $P_{\text{gen}}$ $(H' \leftrightarrow H)$

$v' \gg v \to 0$

$$\bar{\theta} = \sum_{n=1} C_n \left(\frac{v'}{M_q}\right)^{2n}$$

▶ $\mathrm{Im}\ \mathrm{Tr}\left(A^a_q\,[A^b_{q'}\,,A^c_{q''}]\right) f\left(M^a_q, M^b_{q'}, M^c_{q''}\right)$ $\quad (A^a_q)^{ij} \equiv x^{ia}_q x^{\dagger aj}_q$

totally antisymmetric

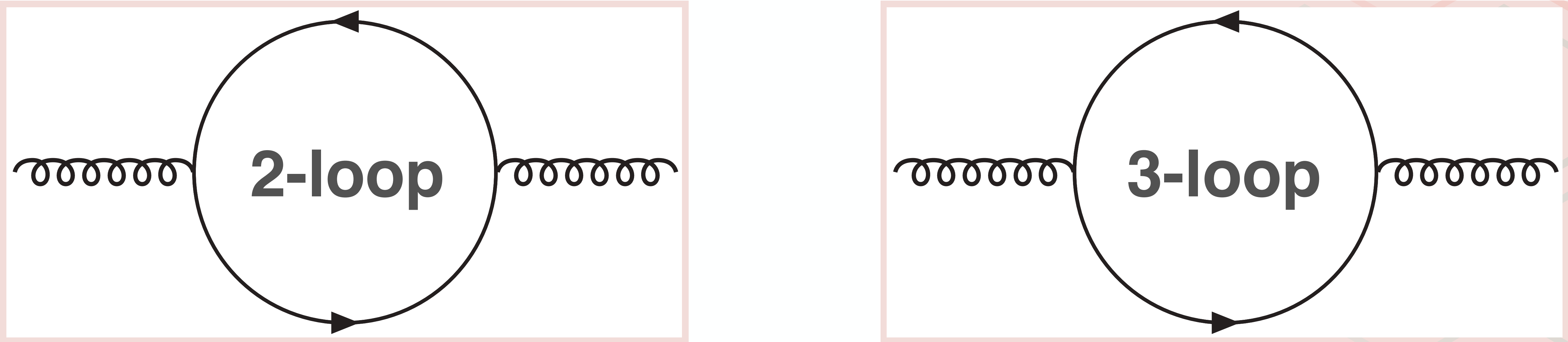
- no SM quarks
- totally antisymmetric

# Non-vanishing radiative $\bar{\theta}$ corrections

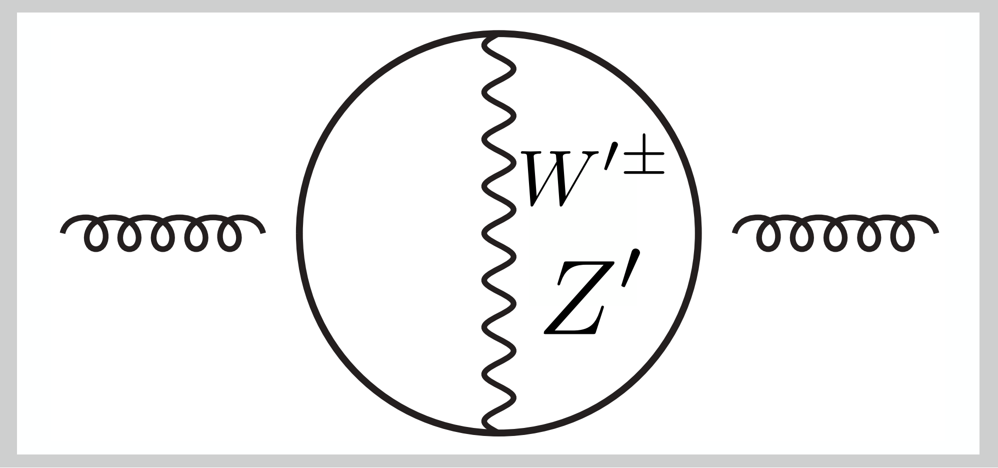
$$
\begin{aligned}
\mathcal{O}(x^4)\cdots\ \mathrm{Im}\ \mathrm{Tr}\left(A_q^a A_{q'}^b\right) f(M_q^a, M_{q'}^b) &= \frac{1}{2}\,\mathrm{Im}\ \mathrm{Tr}\left(A_q^a A_{q'}^b\right) f(M_q^a, M_{q'}^b) - \frac{1}{2}\,\mathrm{Im}\ \mathrm{Tr}\left(A_{q'}^b A_q^a\right) f(M_q^a, M_{q'}^b) \\
&= \frac{1}{2}\,\mathrm{Im}\ \mathrm{Tr}\left(A_q^a A_{q'}^b - A_q^a A_{q'}^b\right) f(M_q^a, M_{q'}^b) \\
&= 0
\end{aligned}
$$

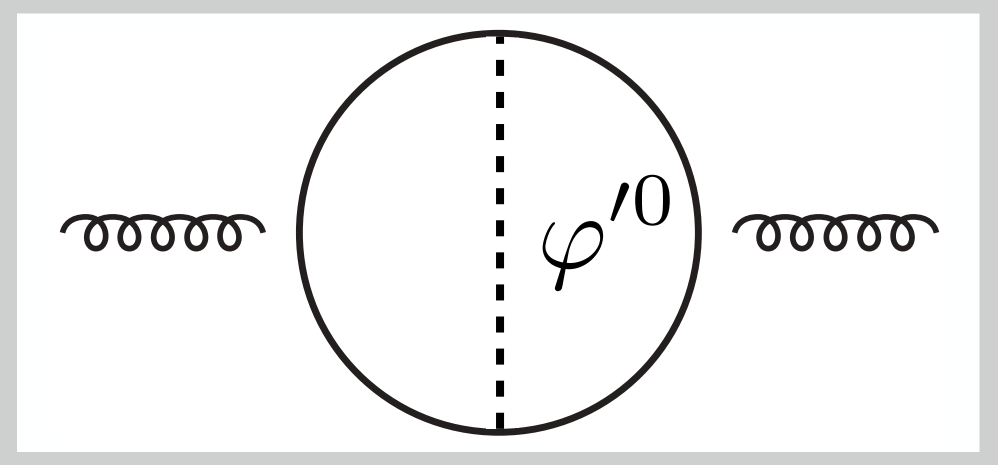
- $f$ is a real loop function
- cyclicity in $\mathrm{Tr}$

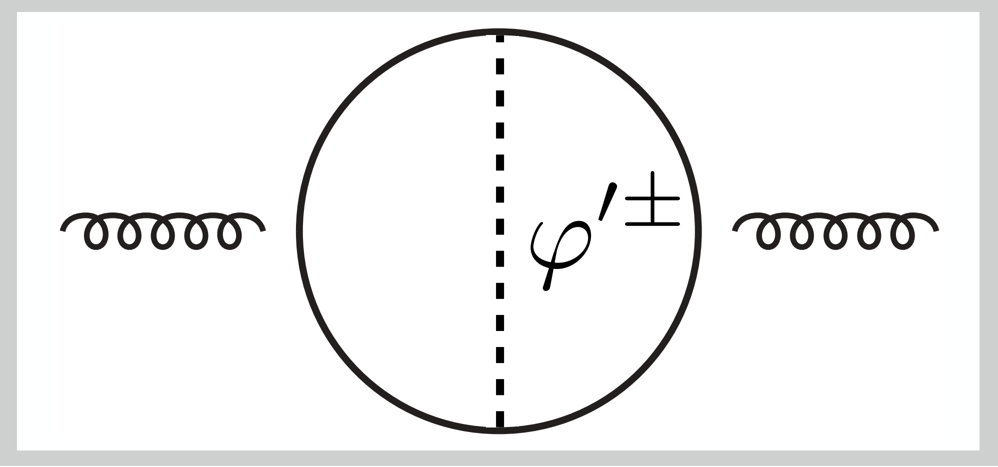
**non-vanishing:** $\mathrm{Im}\ \mathrm{Tr}\left(A_q^a\left[A_{q'}^b\,, A_{q''}^c\right]\right) f(M_q^a, M_{q'}^b, M_{q''}^c)$



# 2-loop contributions



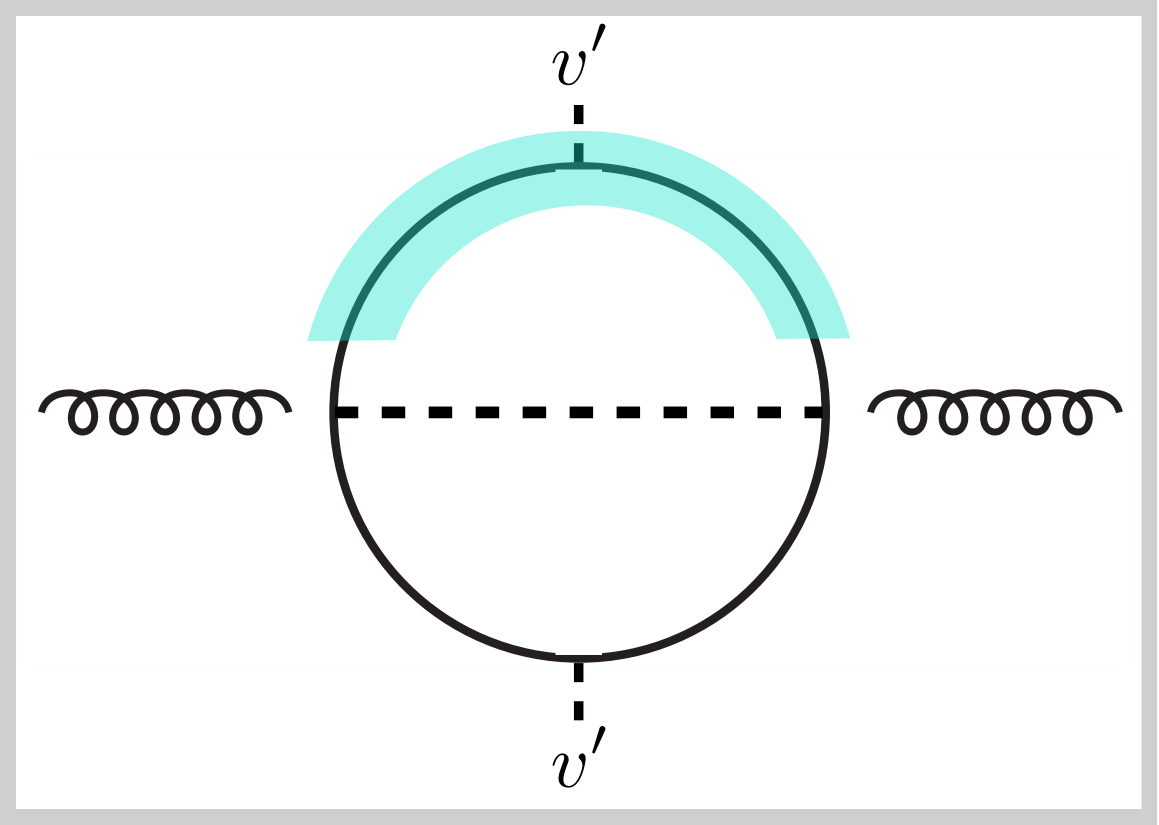




$W'^{\pm}$, $Z'$ : gauge bosons from $SU(2)_R$

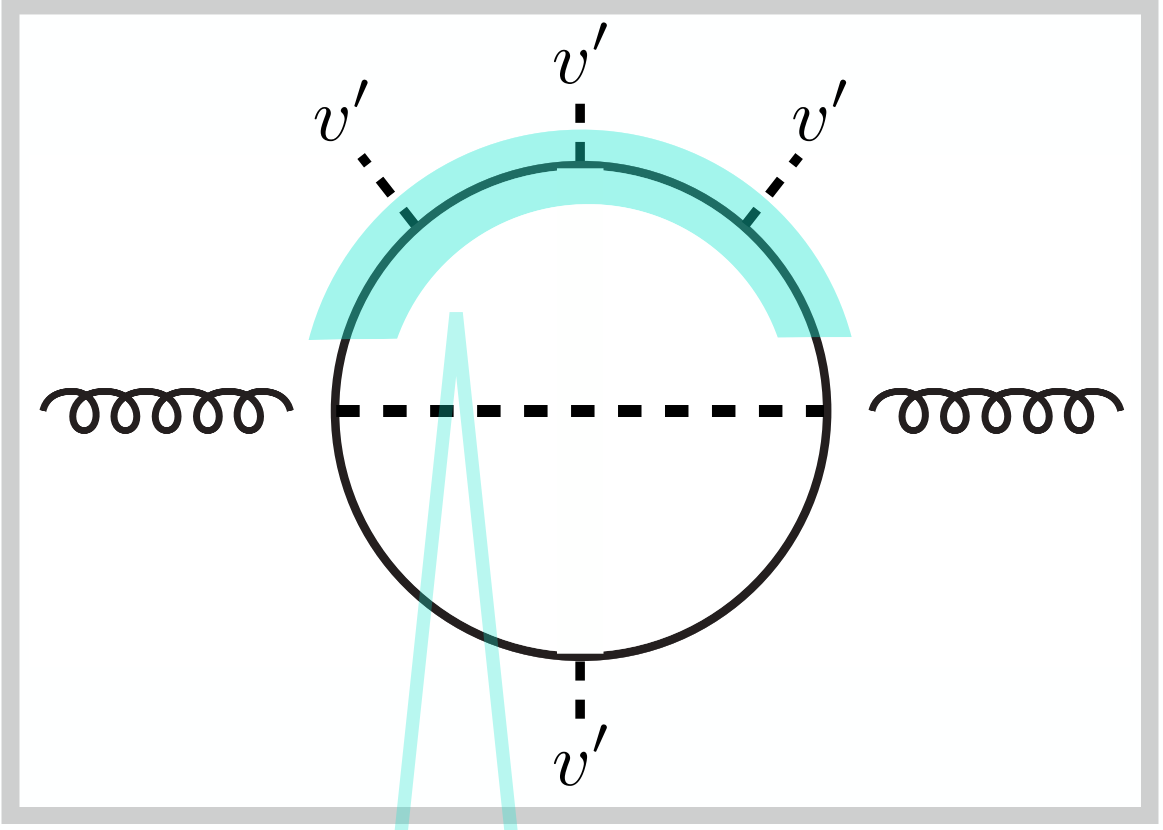
$\varphi'^{0}$, $\varphi'^{\pm}$ : $H'$'s NG bosons

the lowest order: $O(x^4)$




mass insertion

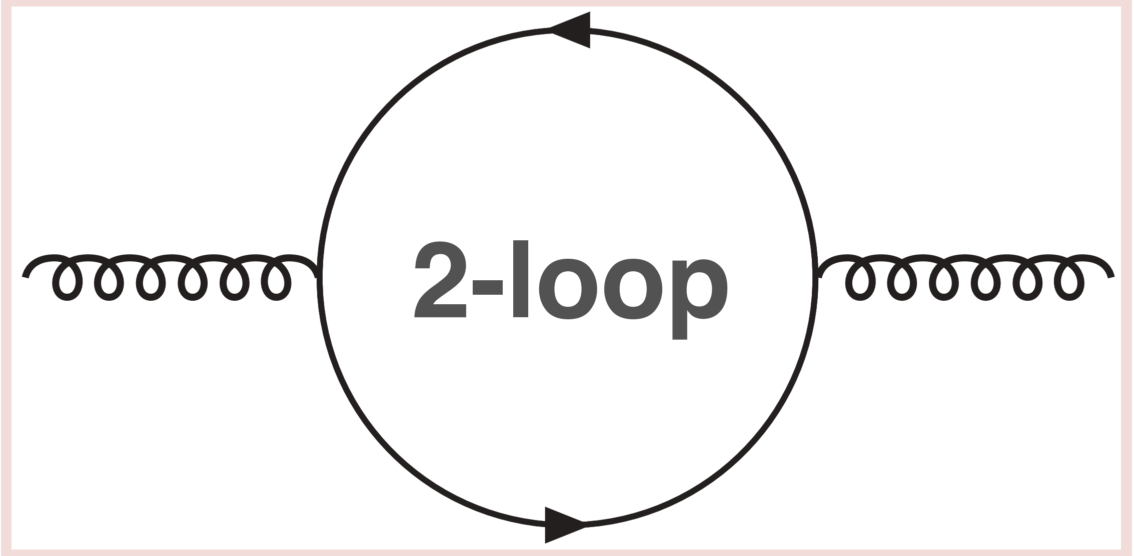
the next order: $O(x^6)$




$$P_L \frac{i(\not{p}+M_u^a)}{p^2-(M_u^a)^2}(-ix_u^{\dagger aj}v'P_R)\frac{i\not{p}}{p^2}(-ix_u^{jb}v'P_L)\frac{i(\not{p}+M_u^b)}{p^2-(M_u^b)^2}(-ix_u^{\dagger bi}v'P_R)\frac{i\not{p}}{p^2}P_L$$

$$= iv'^3 P_L \frac{1}{p^2-(M_u^a)^2}x_u^{\dagger aj}x_u^{jb}\frac{1}{p^2-(M_u^b)^2}x_u^{\dagger bi}$$

**symmetric about $a$ and $b$**



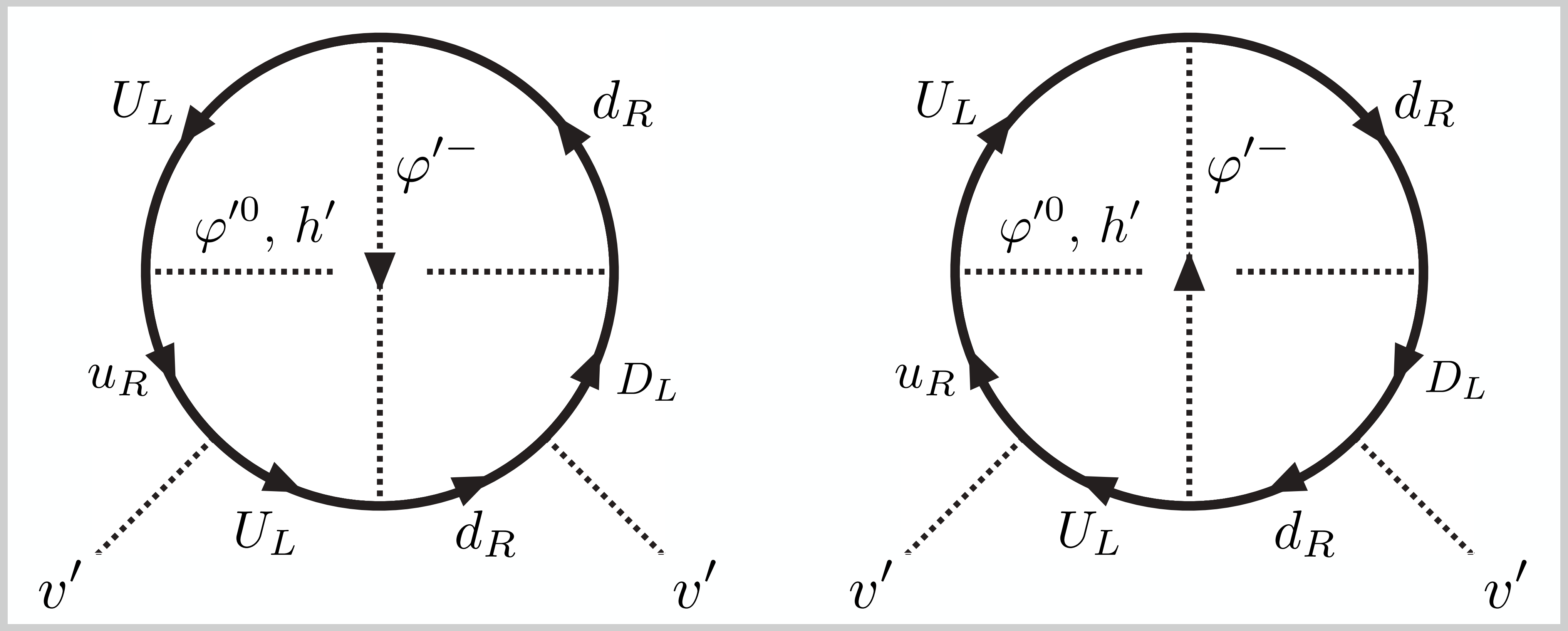

**symmetric**

$$\mathrm{Im}\,\mathrm{Tr}\left(A_q^a\,[A_{q'}^b\,,A_{q''}^c]\right) f(M_q^a, M_{q'}^b, M_{q''}^c)$$

$$=0$$

(analytically & numerically checked!)

# 3-loop contributions




$$(A_q^a)^{ij} \equiv x_q^{ia} x_q^{\dagger aj}$$

$$x_u = V_{\rm CKM}^\dagger \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}\left(\theta_{u3}, \theta_{u8}\right) V_U \frac{\sqrt{M_u}}{\sqrt{v'}}$$

$$x_d = \frac{\sqrt{m_d}}{\sqrt{v}} \frac{\sqrt{M_d}}{\sqrt{v'}}$$

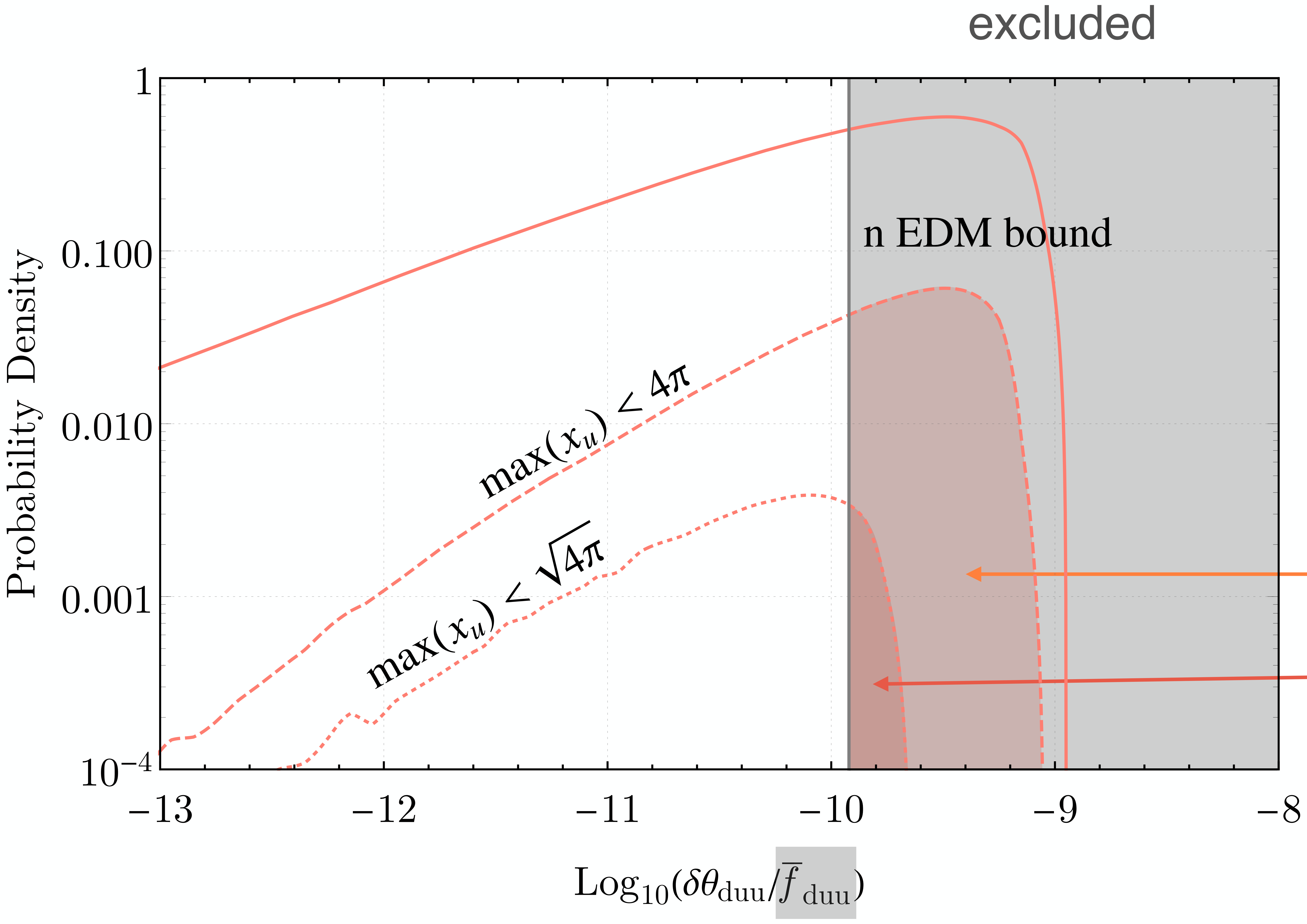
$$\delta\theta_{duu} \approx \frac{1}{(16\pi^2)^2} \frac{v'^2}{\widetilde{M}^2} \operatorname{Im} \operatorname{tr}\left(A_d^a \left[A_u^b, A_u^c\right]\right) f_{duu}^{abc}$$

$$\approx \frac{4}{(16\pi^2)^2} \frac{v'^2}{\widetilde{M}^2} \frac{\widetilde{M}_d M_u^b M_u^c}{v'^3} \frac{m_b m_t^{\frac{3}{2}} \sqrt{m_c}}{v^3} \operatorname{Im}\left(V_{\rm CKM}^{\dagger 33} V_U'^{3b} V_U'^{\dagger b3} V_U'^{3c} V_U'^{\dagger c2} V_{\rm CKM}^{23}\right) \tilde{f}_{duu}^{bc}$$

**free parameters**

- the hierarchy in the VL quark masses $\tilde{M} \equiv M_d^a = M_u^1 \gg M_u^2 \gg M_u^3 = v'$
- angles: 3 mixing angles in $V_U$ + (2+1) $CP$ phases in $\bar{\Phi}$ & $V_U$

# Induced $\bar{\theta}$ in the LR model




including uncertainty of a loop function

**parameter set**

- mass hierarchy in VL quarks
$$\tilde{M} \equiv M_d^a = M_u^1 = M_u^2 = 10^3 M_u^3$$
- 6 angles are taken at random $[0, 2\pi]$

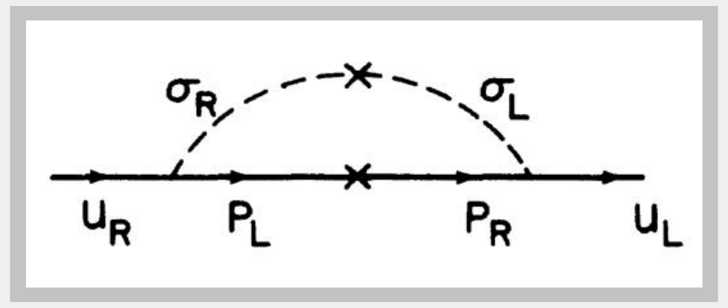
**excluded region ($\bar{f}_{\mathrm{duu}} = 1$)**

| | |
|---|---|
| $\max(x_u) < 4\pi$ | 58.9% |
| $\max(x_u) < \sqrt{4\pi}$ | 10.6% |

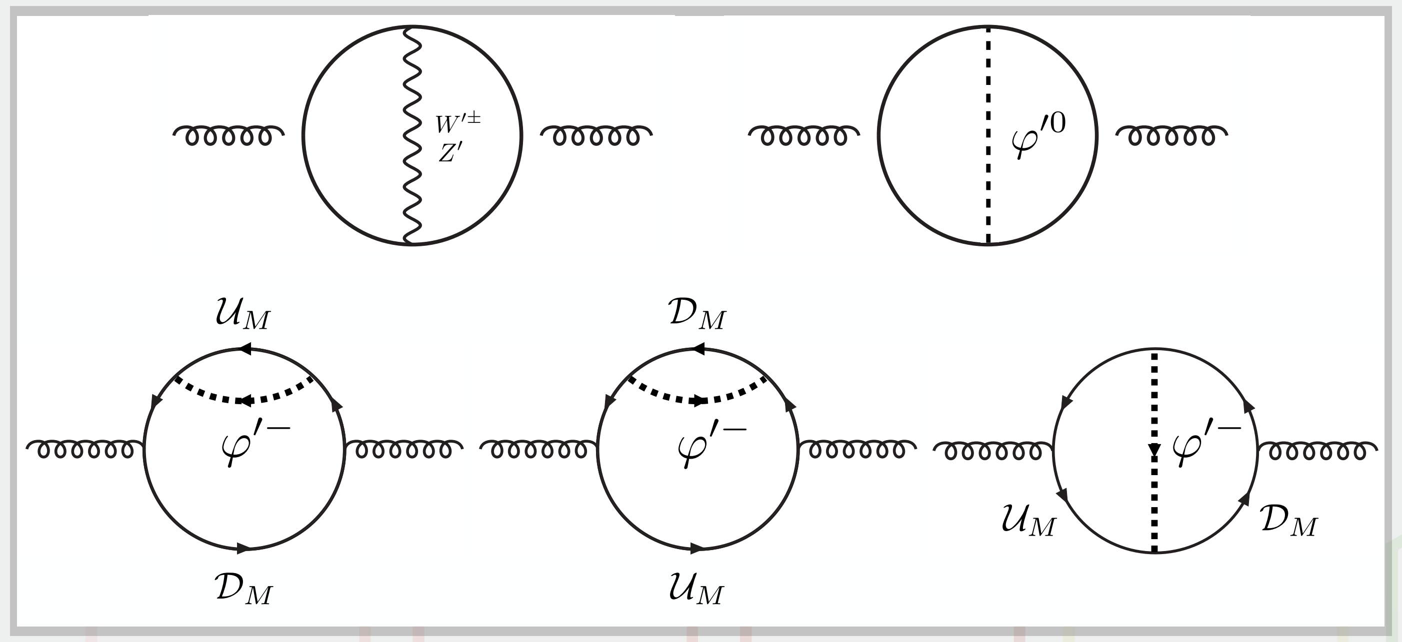
**This model can be probed by a little experimental improvement!**

# 2nd result

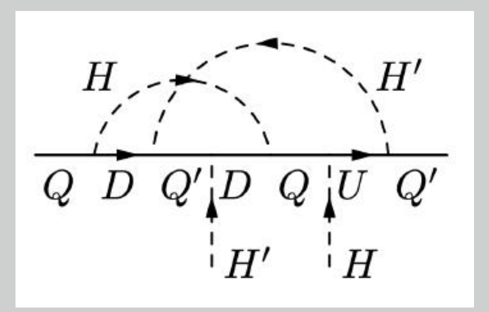
| a mass matrix in the LR model | corrections to $\bar{\theta}$ | **the diagrammatic method** |
|---|---|---|
| tree | ✕ | |
| 1-loop corrections | ✕ | $= 0$ |
| 2-loop corrections | up to a loop function | |



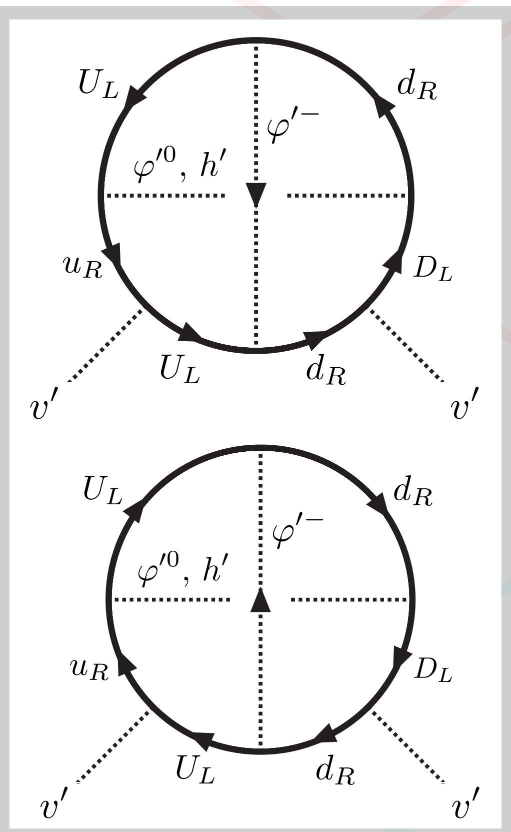

K. S. Babu, R. N. Mohapatra, Phys. Rev. D **41** (1990) 1286

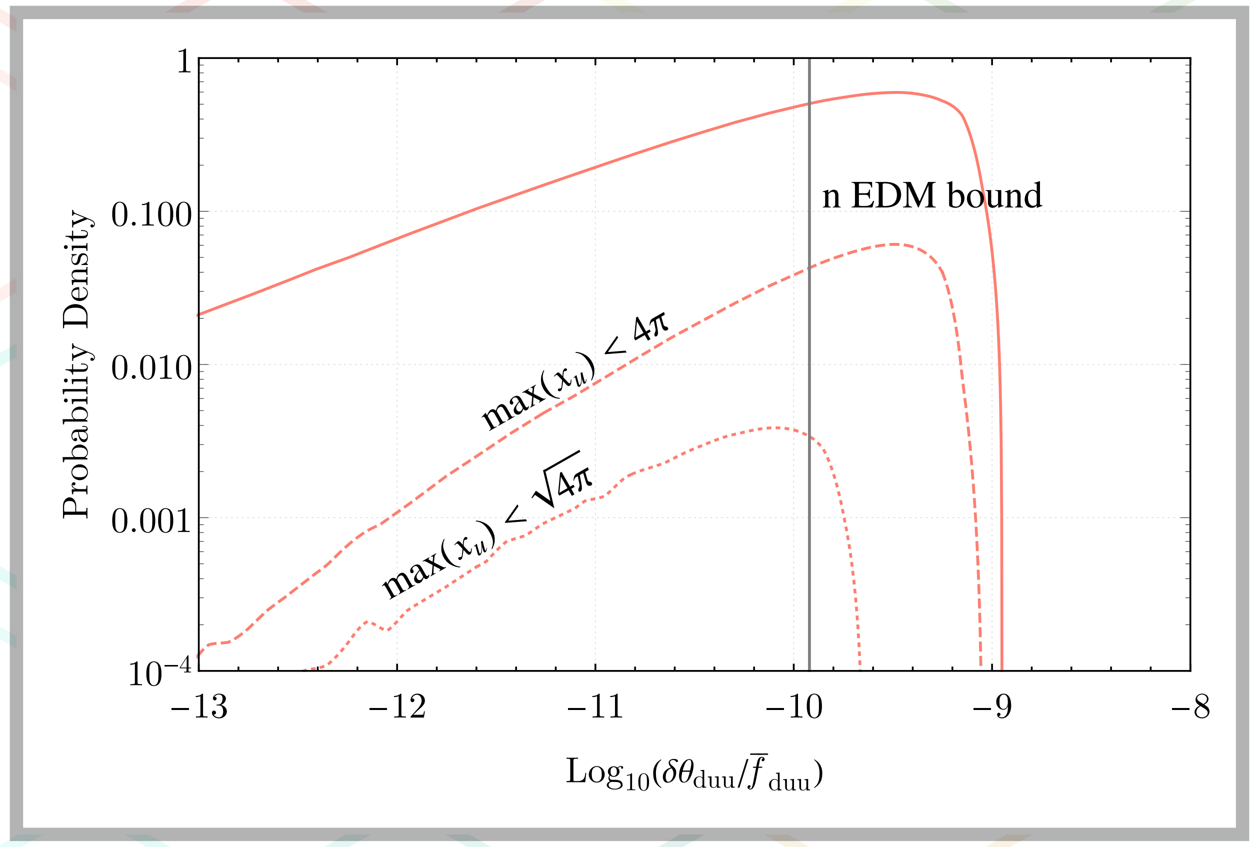



$= 0$




de Vries, et al., arXiv:2109.01630



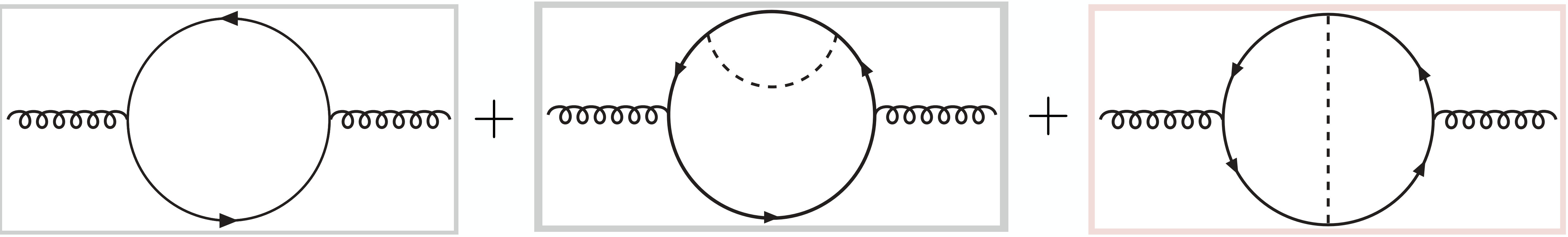



# Contents

# Summary

- **1st result: novel method to calculate radiative corrections to $\bar{\theta}$**

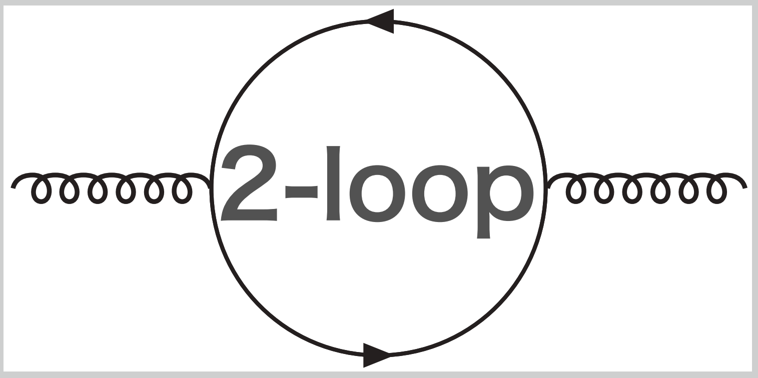
$\bar{\theta} = \theta_G +$



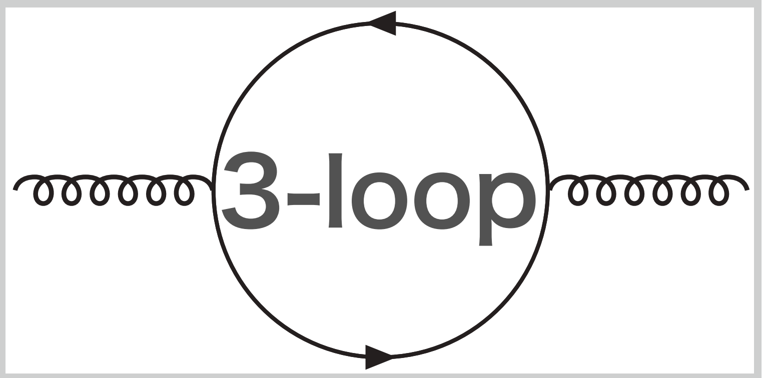
$+ \cdots$

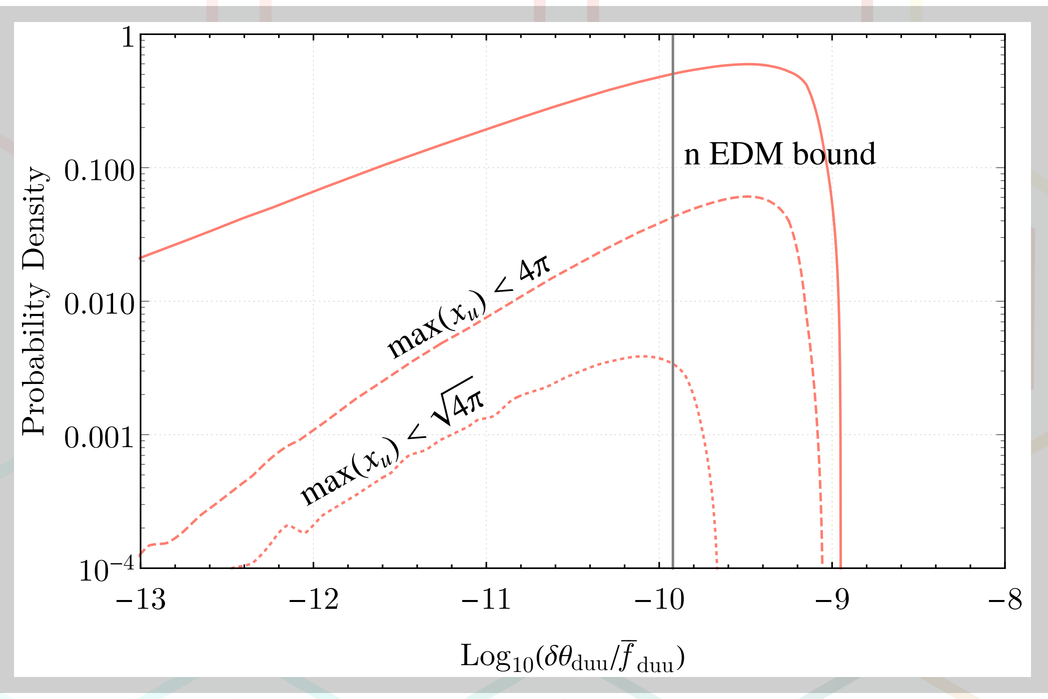
New!

- **2nd result: in the minimal LR model,**




$= 0$




high sensitivity!

# Backup

# axion and quantum gravity

**Peccei-Quinn mechanism** : SM × global symmetry $U(1)_{\rm PQ}$

$$\mathcal{L} \ni \left(\bar{\theta} - \frac{a}{f_a}\right)\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} \xrightarrow[\langle a \rangle = f_a \bar{\theta}]{} 0$$

$a$ :axion field

The $U(1)_{\rm PQ}$ symmetry has to be exact.

R. D. Peccei, H. R. Quinn, Phys. Rev. **38** (1977) 1440-1443

$\Updownarrow$

The quantum gravity imposes a global symmetry does not exist.

Y. Zeldovich, Phys. Lett. A **59** (1976) 254
D. Harlow, H. Ooguri Phys. Rev. Lett. **122** (2019) 191601

# Fujikawa method

$$S_{\mathrm{QCD}} = \int d^4x \left\{ \sum_{f=u,d,s,c,b,t} \bar{f} i \not{D} f - \left( \mathcal{M}_u^{ij} \bar{u}_L^i u_R^j + \mathcal{M}_u^{\dagger ij} \bar{u}_R^i u_L^j + \mathcal{M}_d^{ij} \bar{d}_L^i d_R^j + \mathcal{M}_d^{\dagger ij} \bar{d}_R^i d_L^j \right) \right.$$
$$\left. - \frac{1}{4} G^{\hat{a}}_{\mu\nu} G^{\hat{a}\mu\nu} + \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} \right\}$$

mass diagonalize (chiral rotation)

**—Fujikawa method—**

K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

$$\bar{\theta} = \theta_G - \arg \det \left[ \mathcal{M}_u \mathcal{M}_d \right]$$

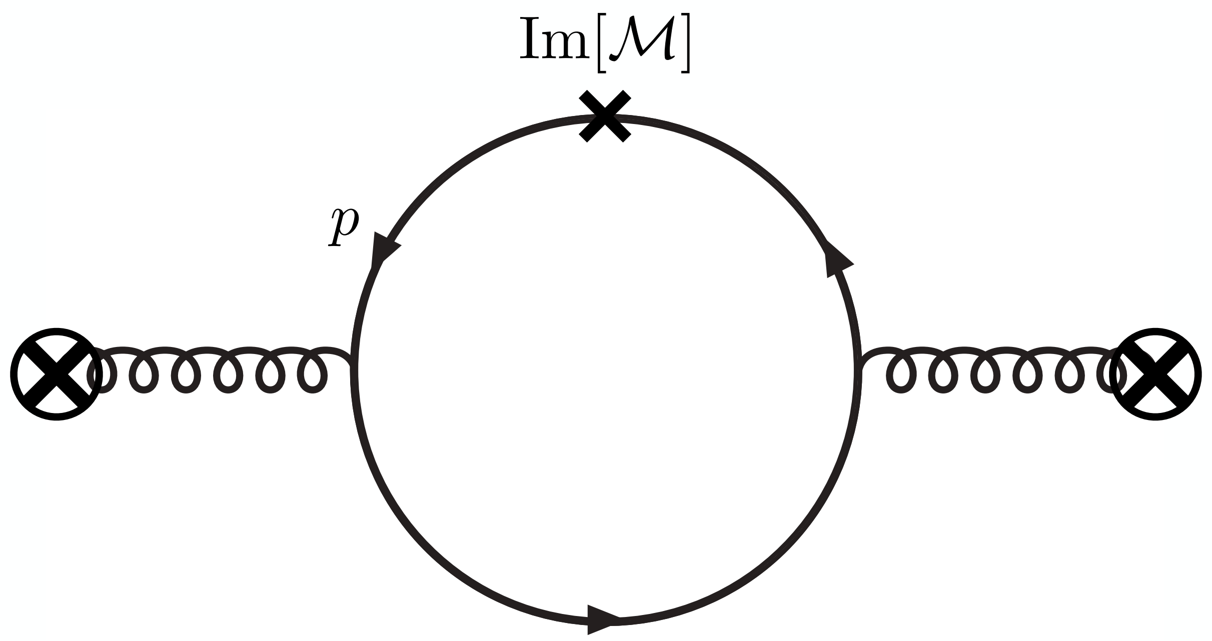
**integrating quarks out**

# Effective aspect of the novel calculation method

$$\bar{\theta} = \theta_G \boxed{- \arg \det \left[ \mathcal{M}_u \mathcal{M}_d \right]}$$

**~~integrating quarks out~~**

**Consistency is checked.**




**integrating quarks out**

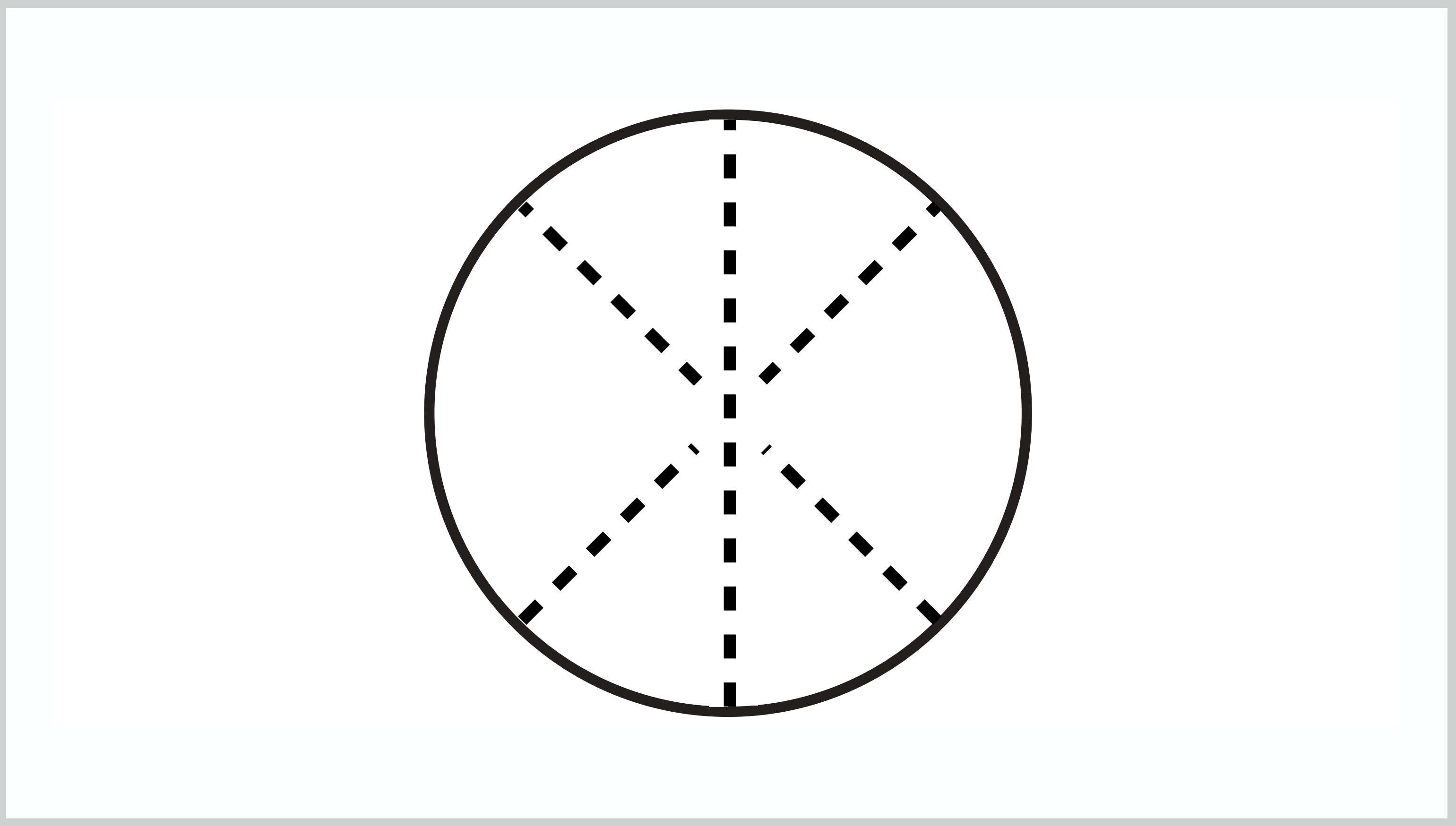
- the perturbative bound: $\mu = \Lambda_{\rm QCD}$
- $m_u \, , m_d \, , m_s < \Lambda_{\rm QCD}$

# Non-vanishing contribution of $\bar{\theta}$

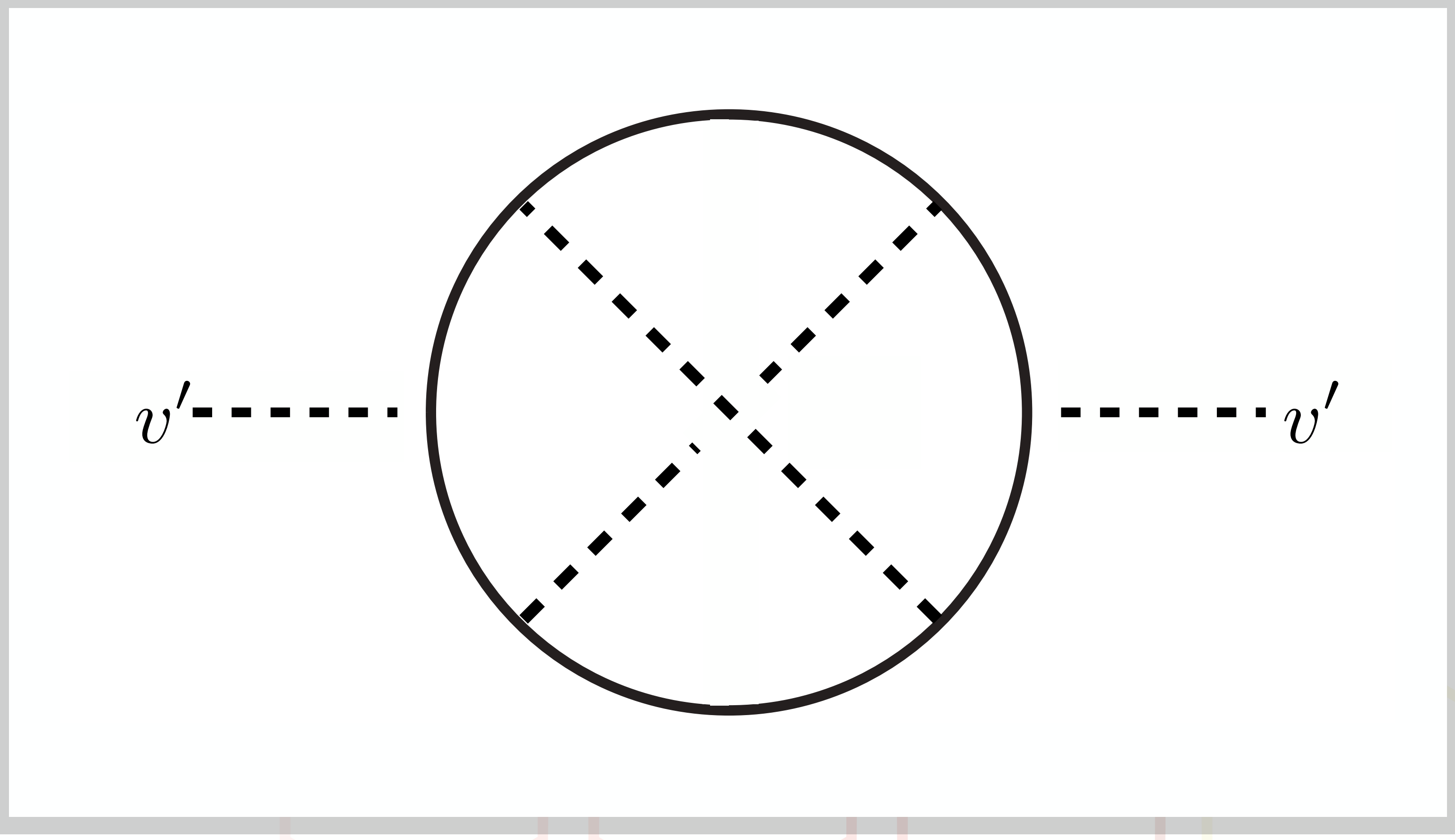
$\mathcal{O}(x^6)$: $\mathrm{Im}\,\mathrm{Tr}\left(A^a_q\left[A^b_{q'},\,A^c_{q''}\right]\right) f\left(M^a_q, M^b_{q'}, M^c_{q''}\right)$

totally antisymmetric

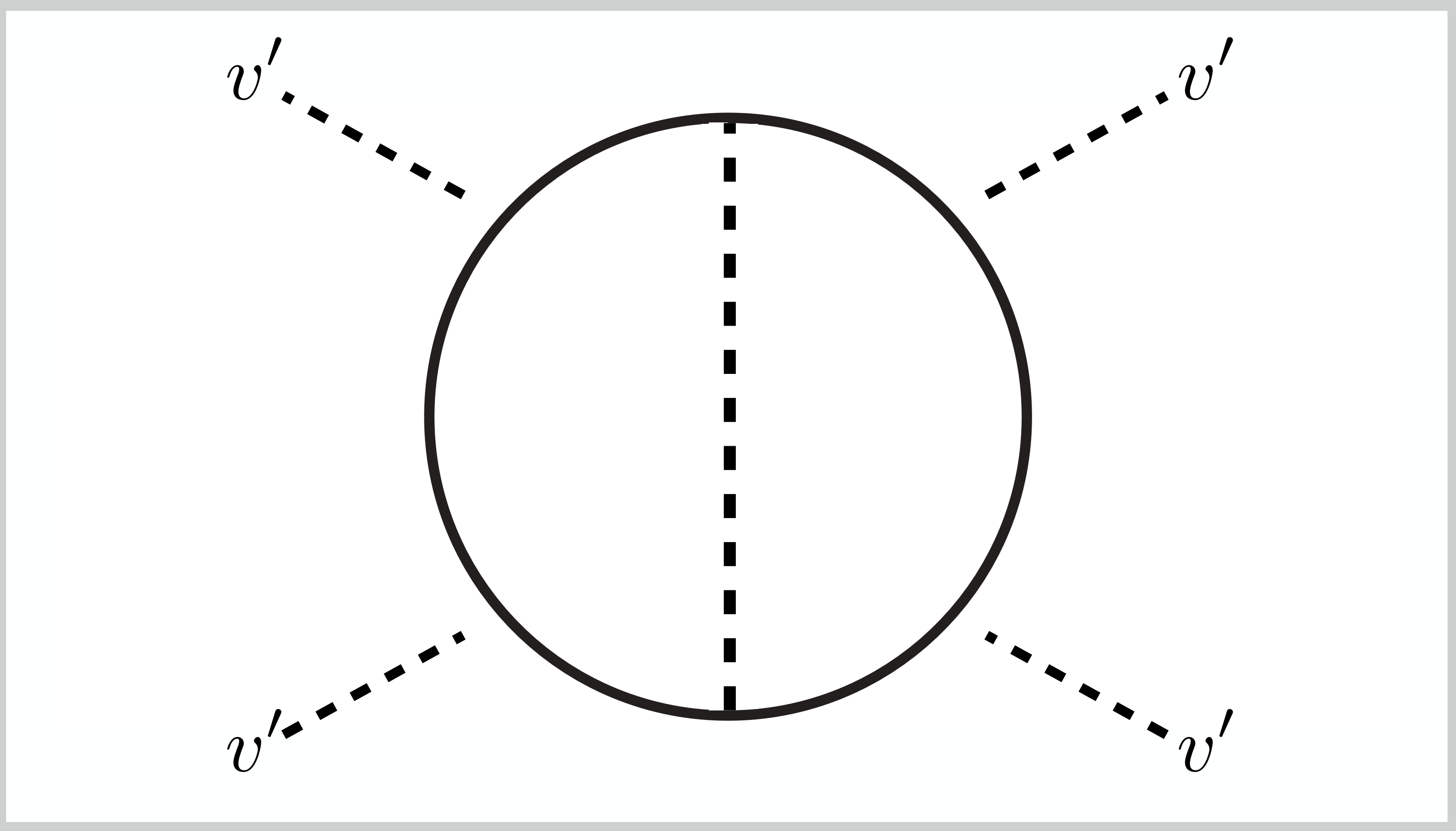
$v'^0$



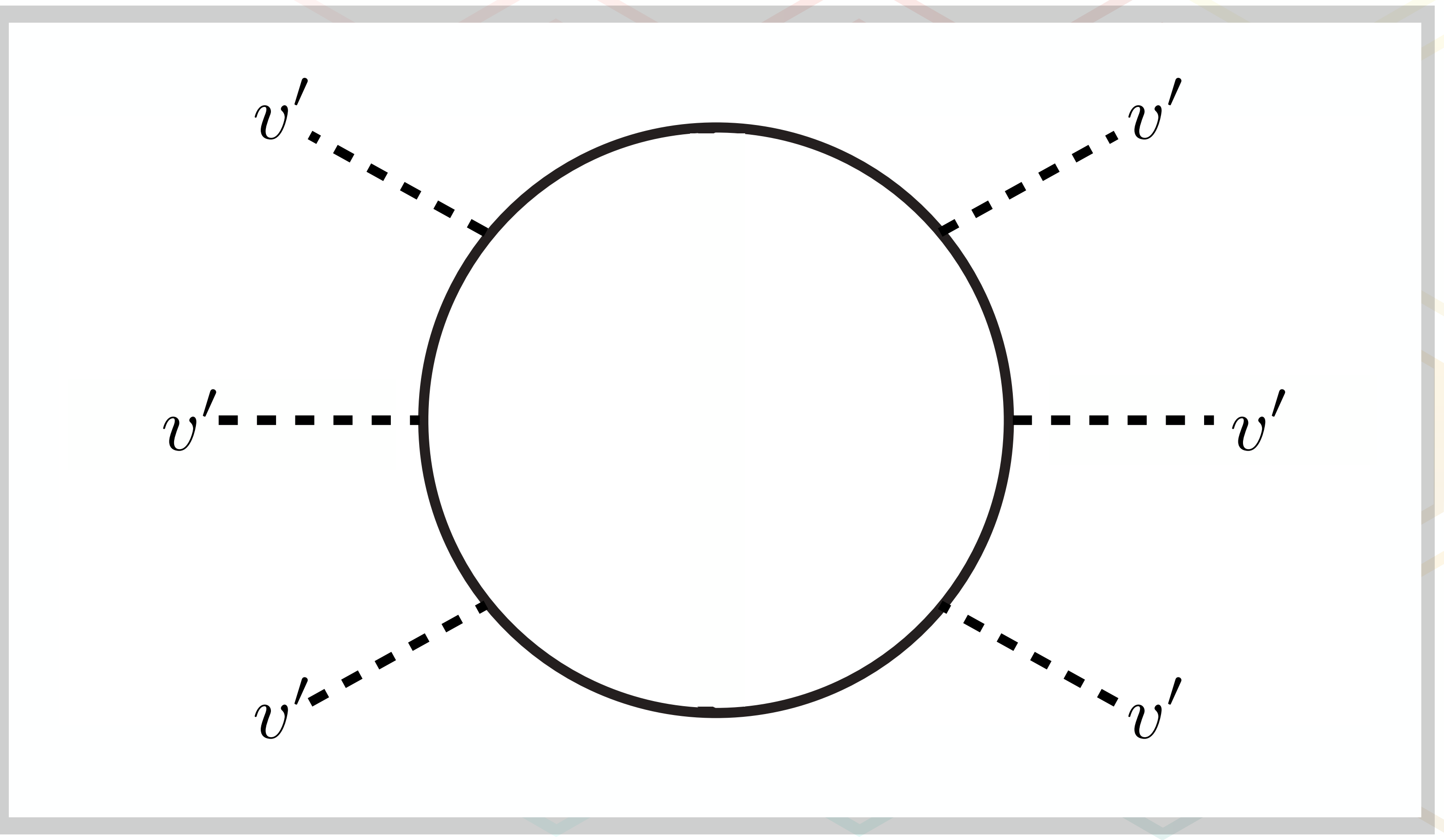
$v'^2$




$v'^4$



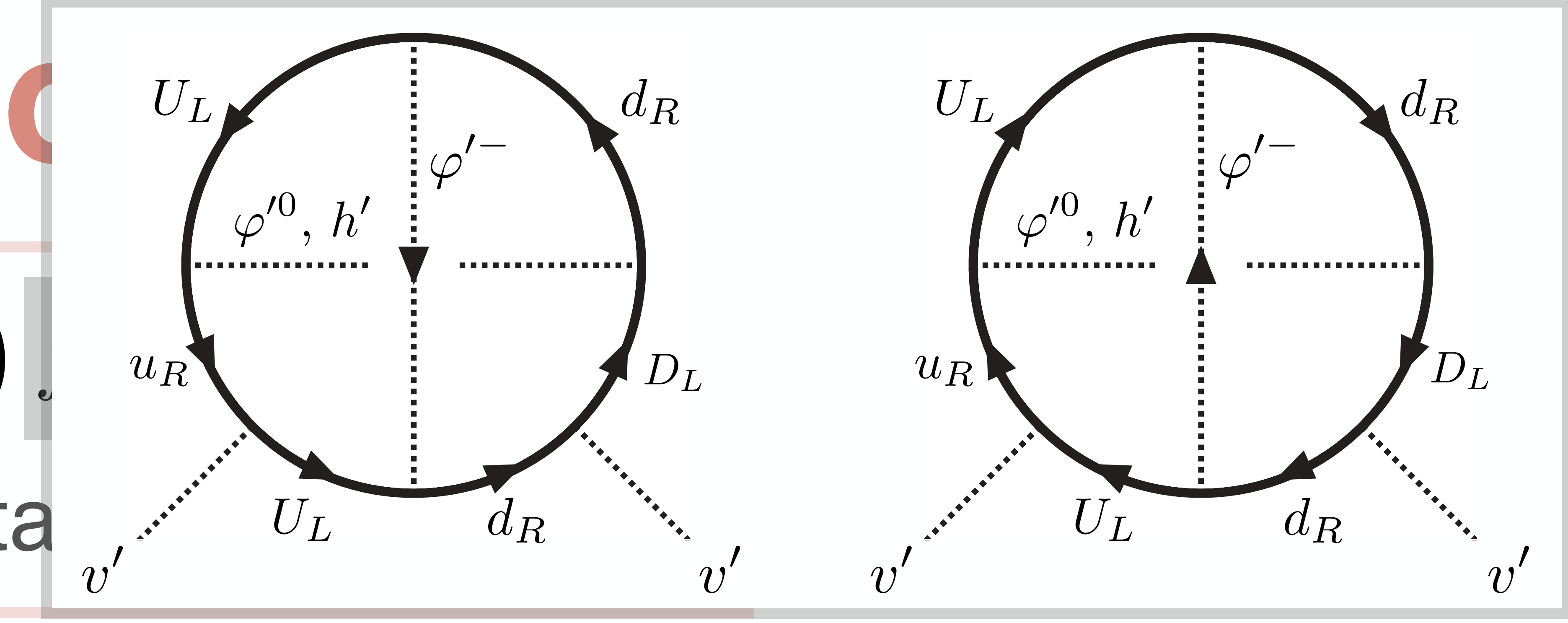

$v'^6$



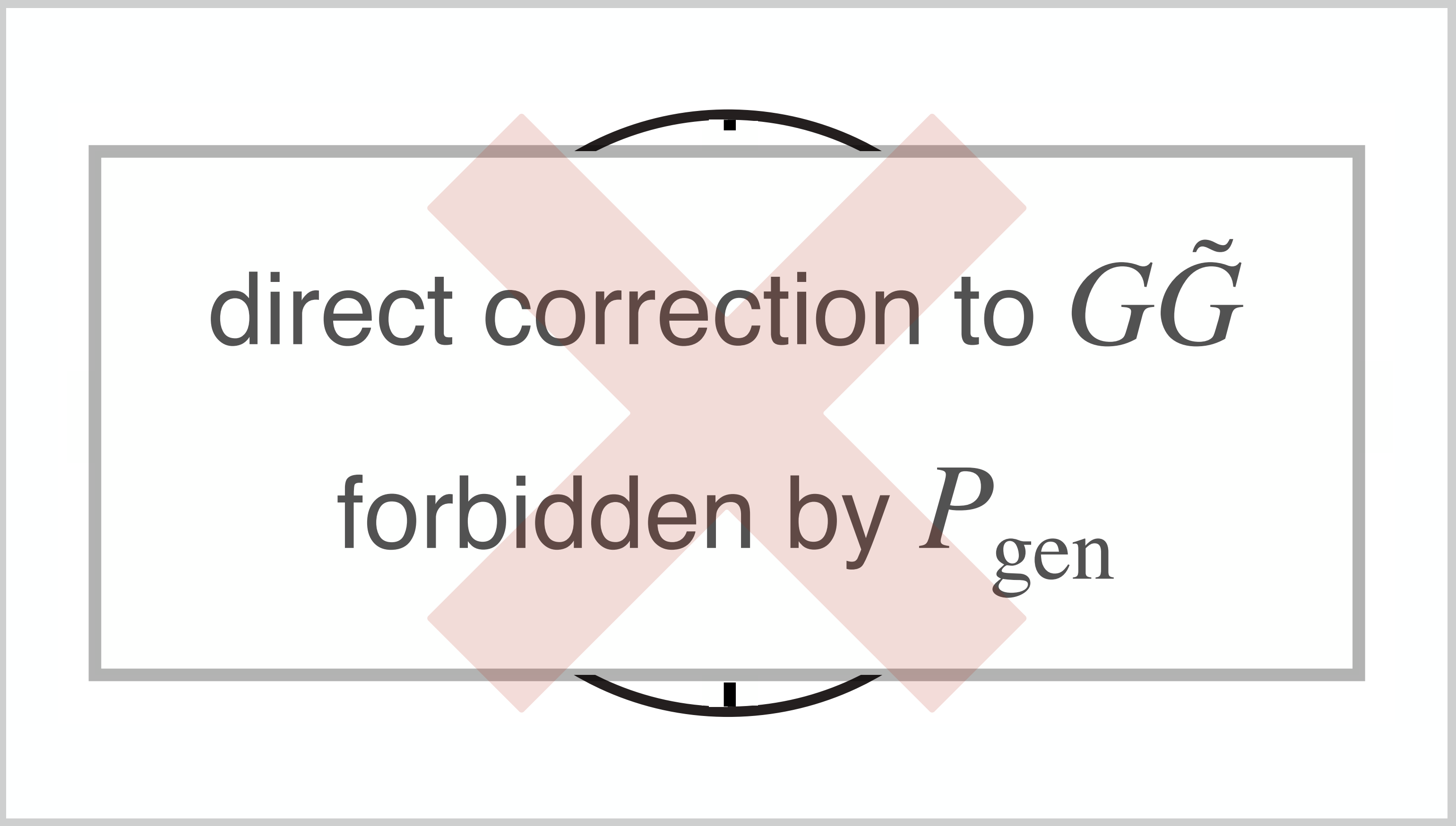
# Non-vanishing contributio

$\mathcal{O}(x^6)$: $\mathrm{Im}\,\mathrm{Tr}\left(A_q^a\left[A_{q'}^b,\,A_{q''}^c\right]\right)$

tota



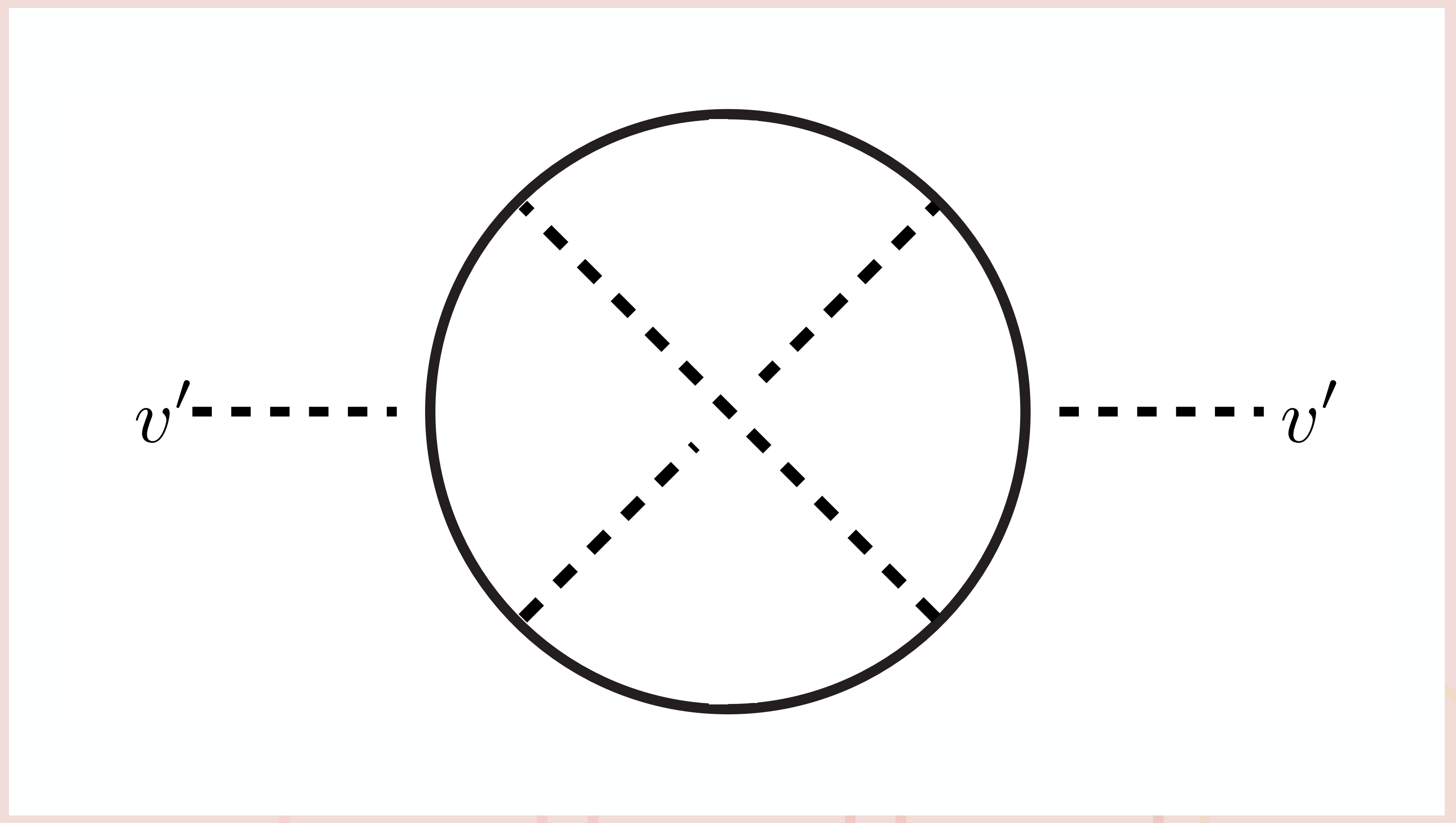

$v'^0$



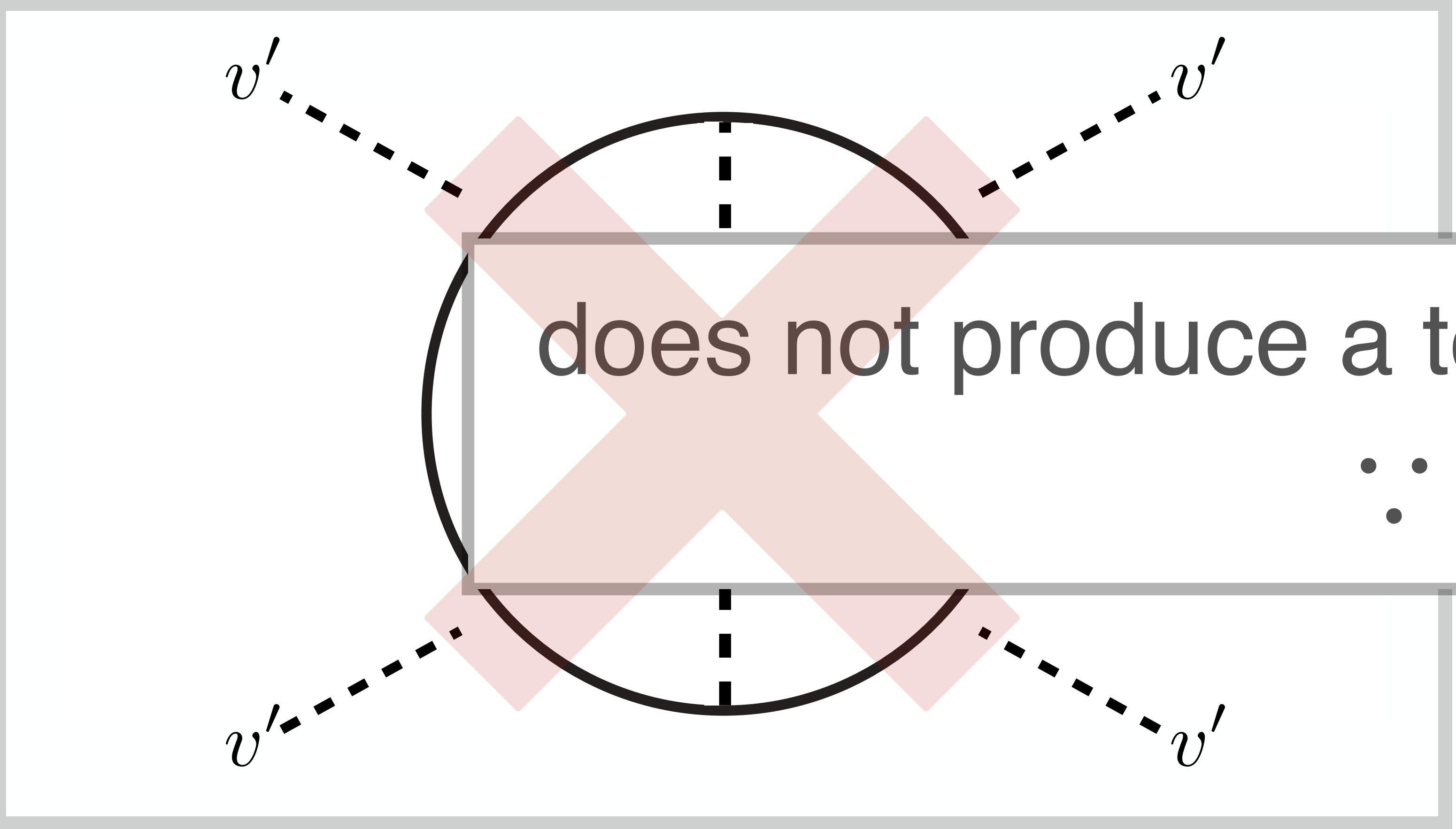
direct correction to $G\tilde{G}$

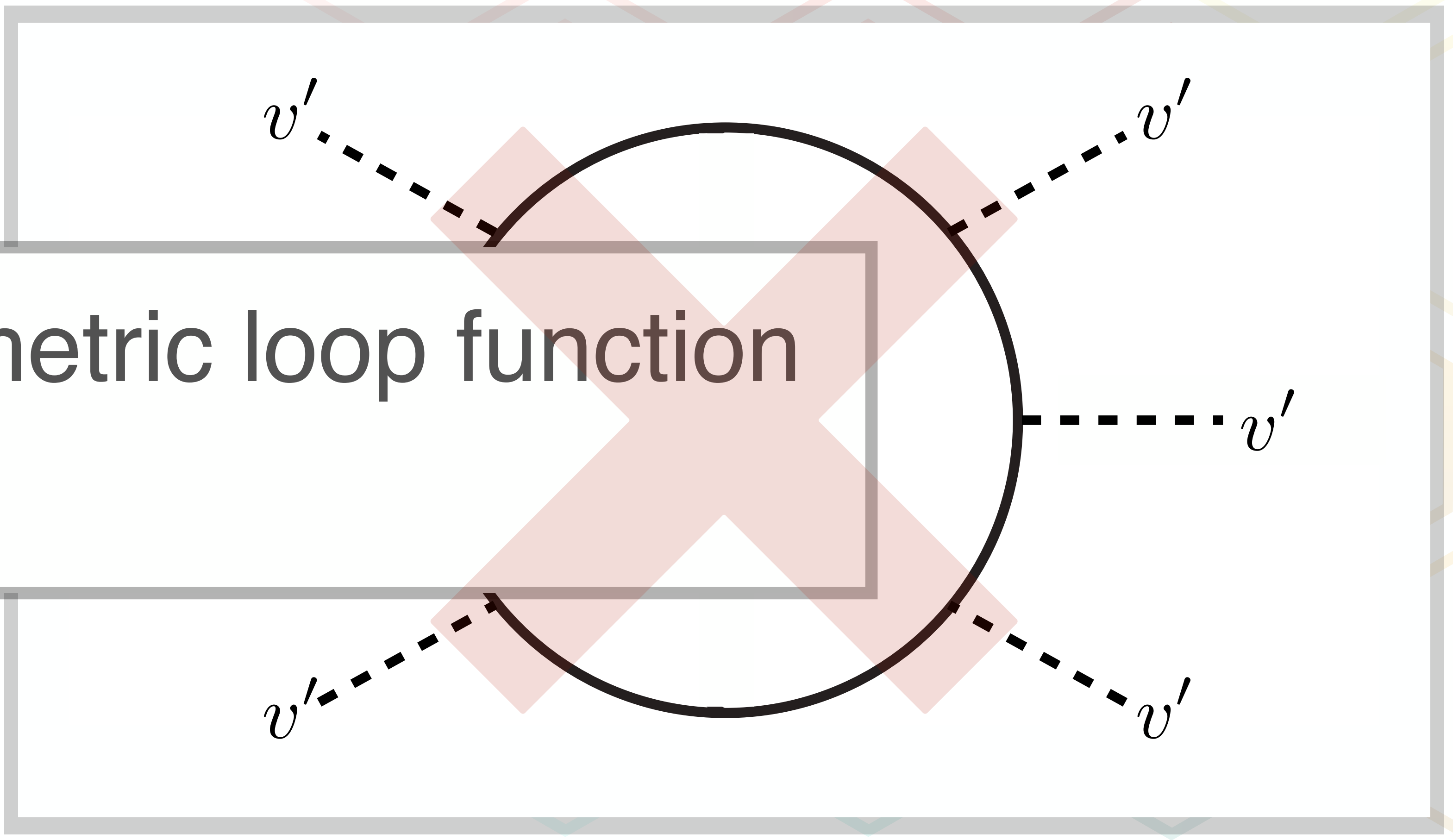
forbidden by $P_{\rm gen}$

$v'^2$




$v'^4$




does not produce a totally antisymmetric loop function

$\because$ mass insertion

# Interactions

gauge
$$\mathcal{L}_{W+W'} = -\frac{g}{\sqrt{2}}\left(\bar{u}^i_L\gamma^\mu d^i_L W^+_\mu + \bar{d}^i_L\gamma^\mu u^i_L W^-_\mu\right) - \frac{g}{\sqrt{2}}\left(\bar{u}^i_R\gamma^\mu d^i_R W'^+_\mu + \bar{d}^i_R\gamma^\mu u^i_R W'^-_\mu\right)$$
$$= -\frac{g}{\sqrt{2}}\left[\left(V^{Pi}_{uL}V^{\dagger iQ}_{uL}\right)\bar{\mathcal{U}}^P_{ML}\gamma^\mu\mathcal{U}^Q_{ML}W^+_\mu + \left(V^{Pi}_{dL}V^{\dagger iQ}_{dL}\right)\bar{\mathcal{D}}^P_{ML}\gamma^\mu\mathcal{D}^Q_{ML}W^-_\mu\right]$$
$$-\frac{g}{\sqrt{2}}\left[\left(V^{Pi}_{uR}V^{\dagger iQ}_{uR}\right)\bar{\mathcal{U}}^P_{MR}\gamma^\mu\mathcal{U}^Q_{MR}W'^+_\mu + \left(V^{Pi}_{dR}V^{\dagger iQ}_{dR}\right)\bar{\mathcal{D}}^P_{MR}\gamma^\mu\mathcal{D}^Q_{MR}W'^-_\mu\right]$$

charged NG
$$-\mathcal{L}_{\varphi^\pm,\varphi'^\pm} = \left(V^{Pi}_{uL}x^{ia}_d V^{\dagger aQ}_{dR}\right)\bar{\mathcal{U}}^P_{ML}\mathcal{D}^Q_{MR}\varphi^+ - \left(V^{Pi}_{dL}x^{ia}_u V^{\dagger aQ}_{uR}\right)\bar{\mathcal{D}}^P_{ML}\mathcal{U}^Q_{MR}\varphi^-$$
$$+\left(V^{Pi}_{uR}x^{ia}_d V^{\dagger aQ}_{dL}\right)\bar{\mathcal{U}}^P_{MR}\mathcal{D}^Q_{ML}\varphi'^+ - \left(V^{Pi}_{dR}x^{ia}_u V^{\dagger aQ}_{uL}\right)\bar{\mathcal{D}}^P_{MR}\mathcal{U}^Q_{ML}\varphi'^-$$
$$+\,\mathrm{h.c.}$$

neutral NG
$$-\mathcal{L}_{\varphi^0,\varphi'^0} = \frac{i}{\sqrt{2}}\left(V^{Pi}_{uL}x^{ia}_u V^{\dagger aQ}_{uR}\right)\bar{\mathcal{U}}^P_{ML}\mathcal{U}^Q_{MR}\varphi^0 - \frac{i}{\sqrt{2}}\left(V^{Pi}_{dL}x^{ia}_d V^{\dagger aQ}_{dR}\right)\bar{\mathcal{D}}^P_{ML}\mathcal{D}^Q_{MR}\varphi^0$$
$$+\frac{i}{\sqrt{2}}\left(V^{Pi}_{uR}x^{ia}_u V^{\dagger aQ}_{uL}\right)\bar{\mathcal{U}}^P_{MR}\mathcal{U}^Q_{ML}\varphi'^0 - \frac{i}{\sqrt{2}}\left(V^{Pi}_{dR}x^{ia}_d V^{\dagger aQ}_{dL}\right)\bar{\mathcal{D}}^P_{MR}\mathcal{D}^Q_{ML}\varphi'^0$$
$$+\,\mathrm{h.c.}$$

# Mass spectrum

- up-type VL quark mass hierarchy $\longrightarrow$ up-type quark mass hierarchy

$$M_u^1 \gg M_u^2 \gg M_u^3$$

- down-type Yukawa ($x_d$) components $\longrightarrow$ down-type quark mass hierarchy ($\because$ mild)

$$M_d^1 = M_d^2 = M_d^3$$

energy scale

$$M_u^1 \sim M_d^a \sim \tilde{M} \gg 100v'$$

universal

$$M_u^2$$

TeV

$$m_{W'} \sim m_{Z'} \sim m_{h'} \sim v' \sim M_u^3$$

**ATLAS**

$$m_{W'} \gtrsim 6\mathrm{TeV}, \quad v' \gtrsim 18\mathrm{TeV}$$

$100$ **GeV**

$$m_t \sim m_h \sim v$$

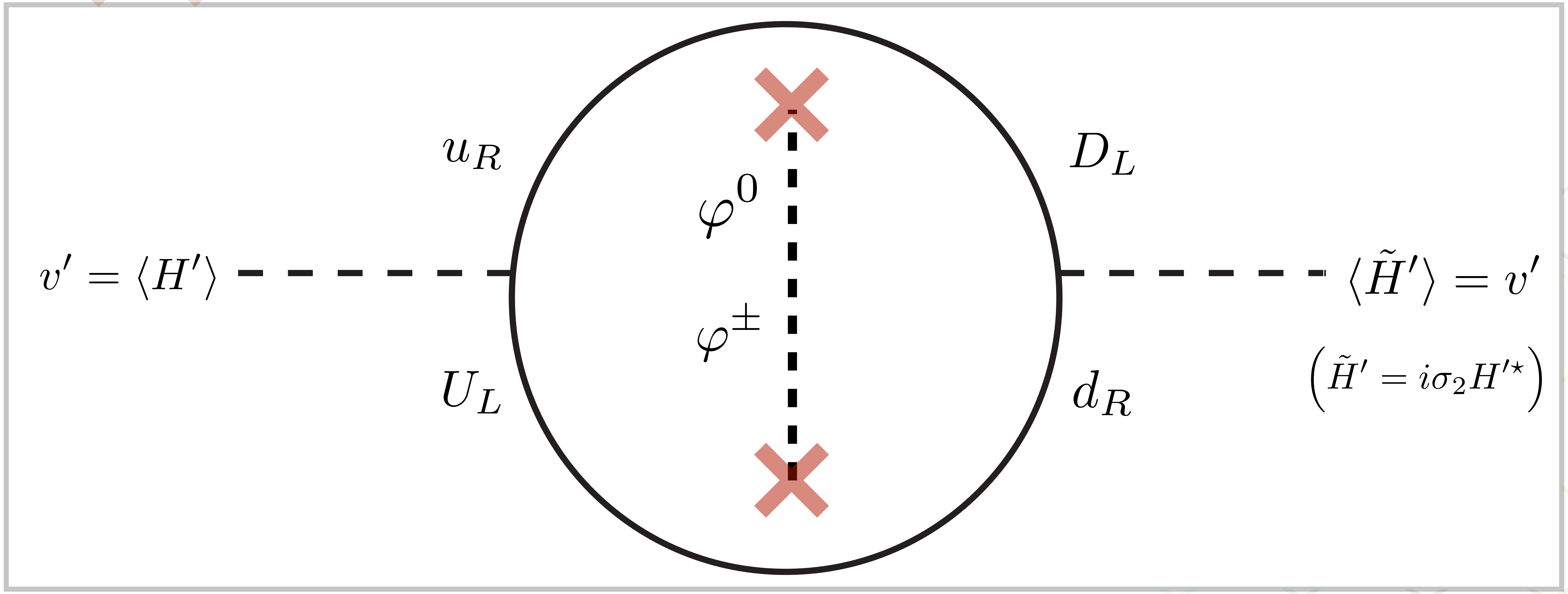
N. Craig, *et al.*, JHEP 09 (2021) 130

# Loops of $SU(2)_L$ Higgs

2-loop

3-loop

$$\frac{|H'|^2 - \cancel{|H|^2}}{\Lambda^2} G\tilde{G} \to \frac{v'^2 - \cancel{v^2}}{\Lambda^2} G\tilde{G}$$



# Collider & flavor experimental bound

- **ATLAS (charged lepton + missing)** $\longrightarrow$ **charged boson mass $m_{W'}$**

$$m_{W'} \gtrsim 6\text{TeV}, \quad v' \gtrsim 18\text{TeV}$$

- **Future Circular Collider (FCC), 100TeV $pp$ collider**

$$m_{W'}, m_{Z'} \sim 40\text{TeV}, \quad v' \gtrsim 120\text{TeV}$$

: fine-tuning problem
in the scalar potential

- **one-loop FCNCs, kaon mixing**

$$(\Delta m_K)_{u,c} \approx -6 \cdot 10^{-16}\text{GeV} \left(\frac{6\text{TeV}}{m_{W'}}\right)^2, \quad |\epsilon_K|_{u,c} \approx 7 \cdot 10^{-5} \left(\frac{6\text{TeV}}{m_{W'}}\right)^2$$

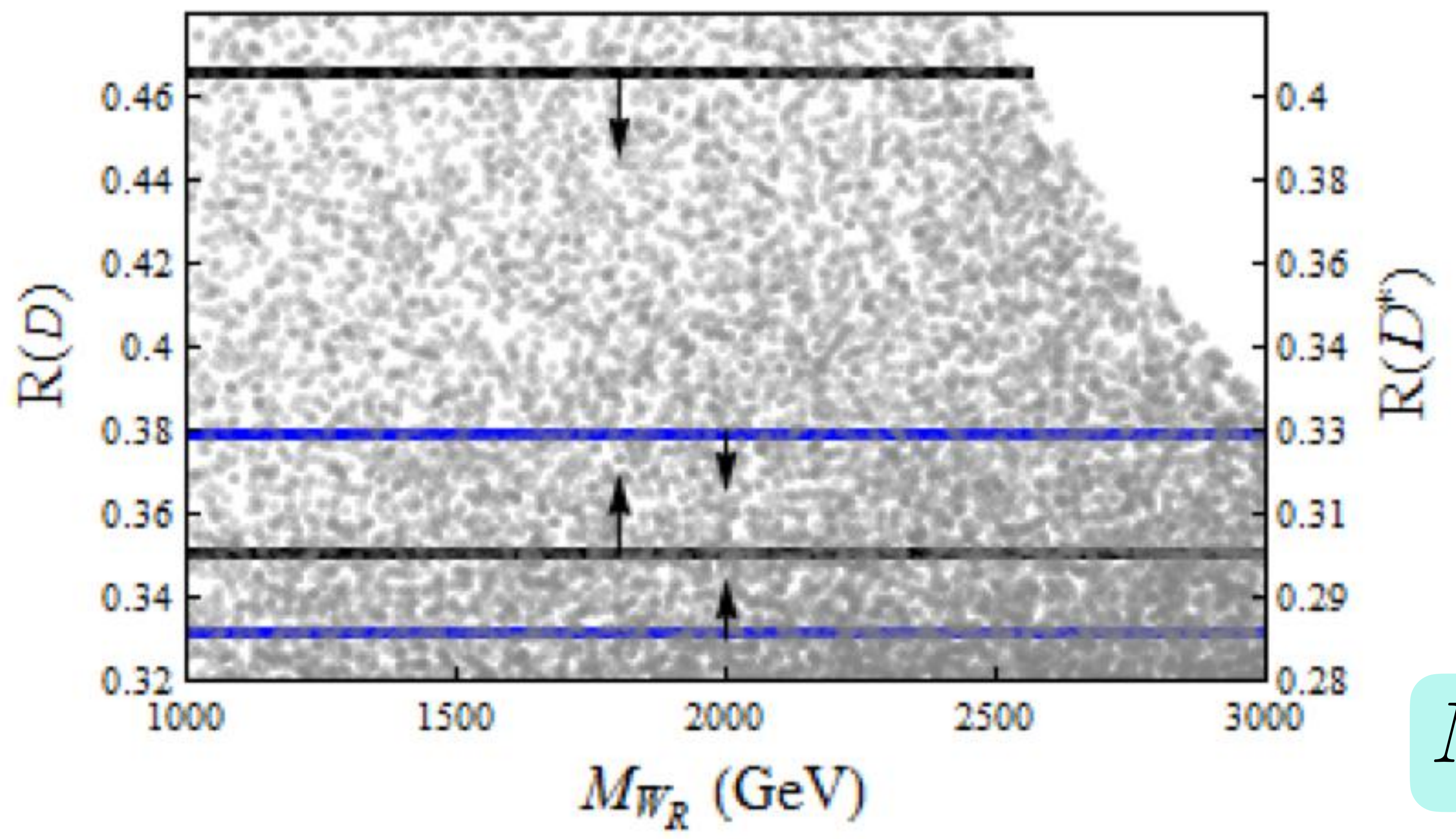
an order of magnitude below the theoretical error in the SM prediction

N. Craig, I. Garcia Garcia, G. Koszegi, and A. McCune, JHEP **09** (2021) 130

# B anomaly in the LR model

$R(D), R(D^*)$ **anomaly**

$$R(D) = \frac{\Gamma(B \to D\tau\nu)}{\Gamma(B \to D\ell\nu)}, \quad R(D^*) = \frac{\Gamma(B \to D^*\tau\nu)}{\Gamma(B \to D^*\ell\nu)}$$




excluded

$M_{W_R} < 3\,\mathrm{TeV}$

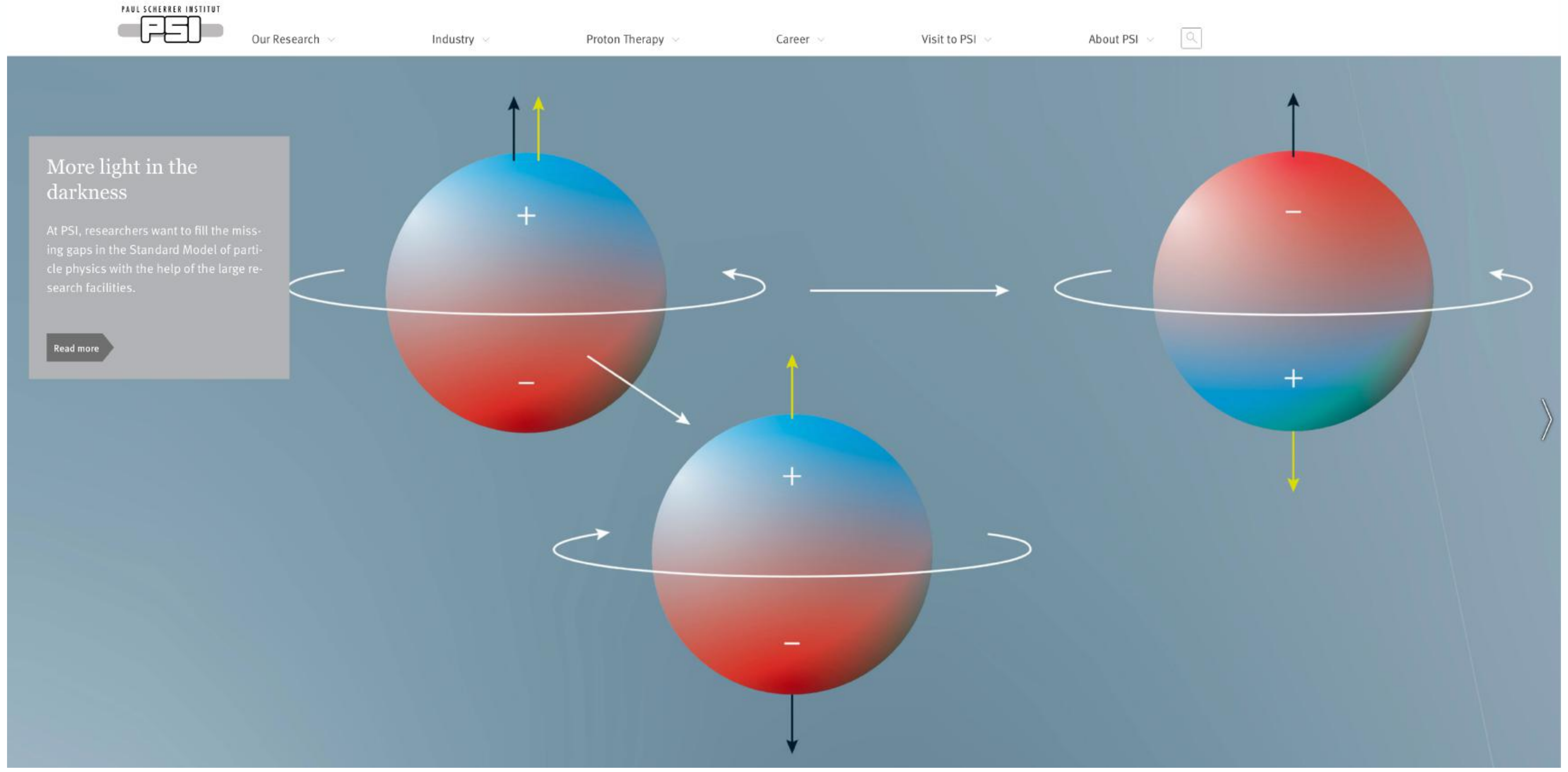
Figure 4: $R(D, D^*)$ scatter-plot is shown by varying $g_R$ and $M_{W_R}$. The boundaries of $R(D)$ and $R(D^*)$ anomalies are shown by black and blue lines respectively. We show 1 $\sigma$ allowed regions.

K. S. Babu, B. Dutta and R. N. Mohapatra, JHEP 01 (2019), 168

# Neutron EDM experiment

## neutron EDM (nEDM) experiment

- Paul-Scherrer Institute (PSI)
- ultracold neutron
  +
  Ramsey method
- result in 2020（measured in 2015 ~ 2016）




https://www.psi.ch/en

$$|d_n| < 1.8 \times 10^{-26} e\,\mathrm{cm} \quad (90\%\mathrm{C.L.})$$

C. Abel, Phys. Rev. Lett. 124 (2020) 081803

**future experiment**: TUCAN(TRIUMF Ultra-Cold Advanced Neutron)

Canada & Japan

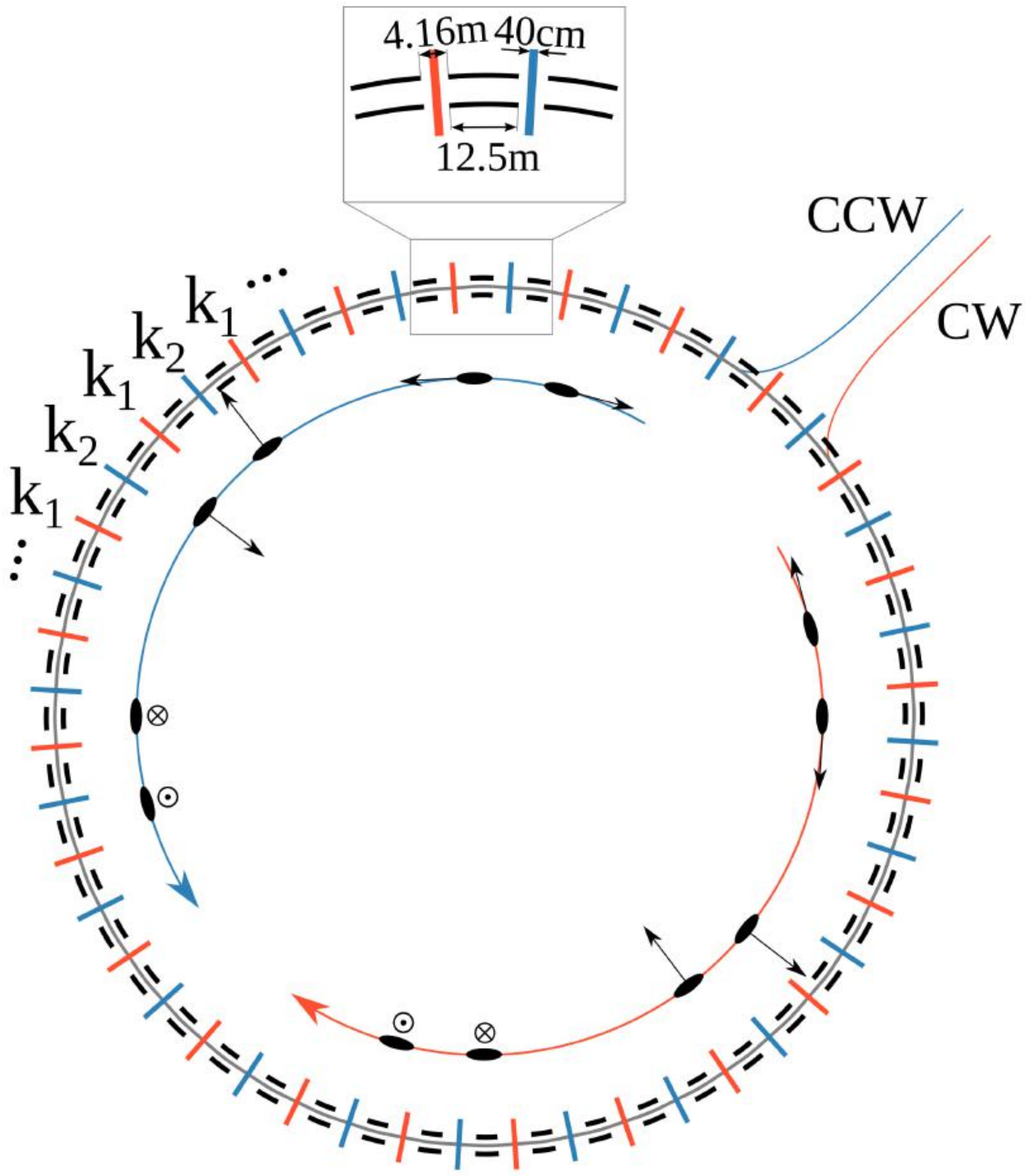
Aim: $|d_n| \lesssim 1 \times 10^{-27} e\,\mathrm{cm}$

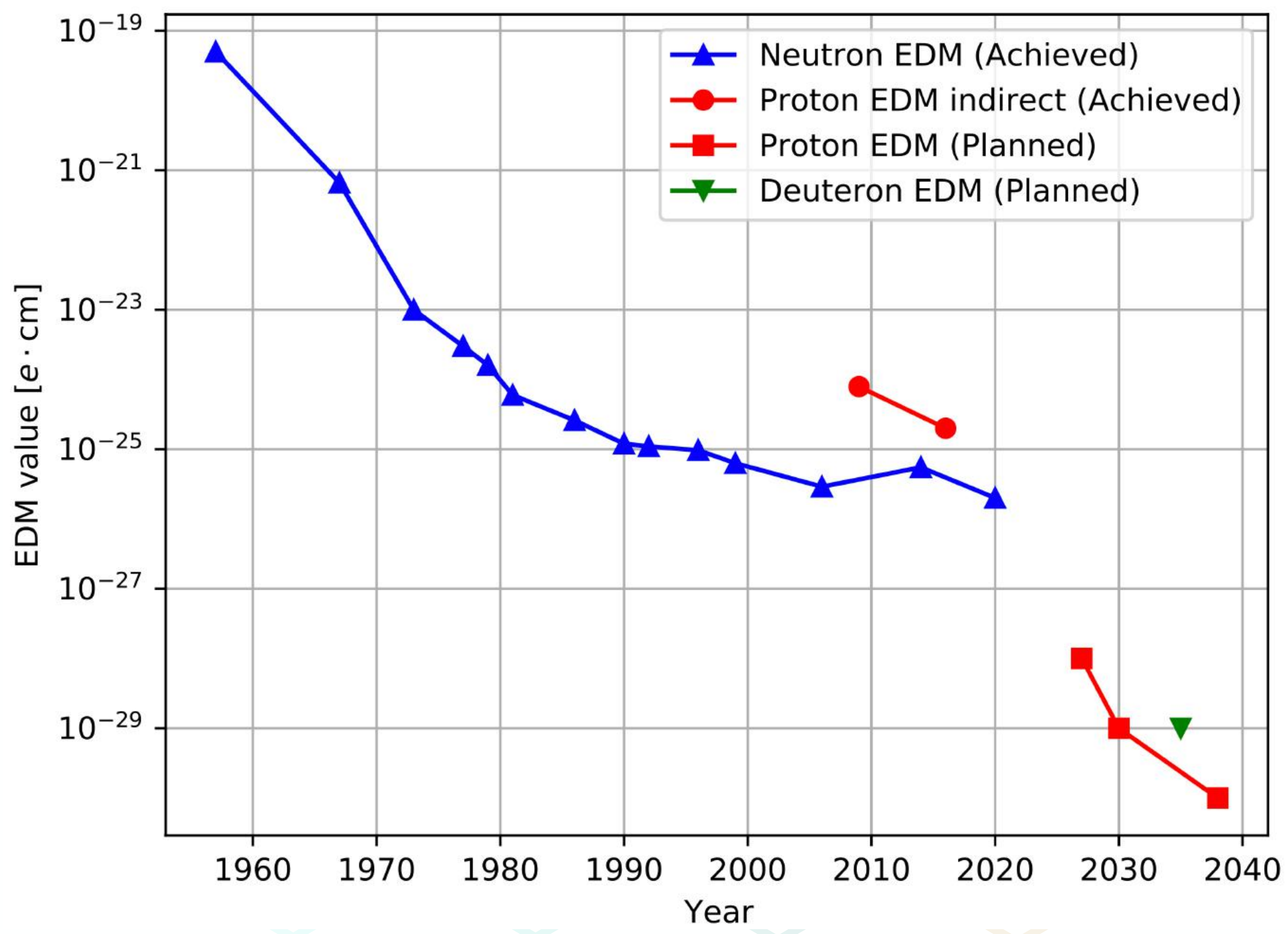
S. Ahmed, *et al.*, *Phys.Rev.C* 99 (2019) 2, 025503

# Proton EDM experiment

## proton EDM (pEDM) experiment

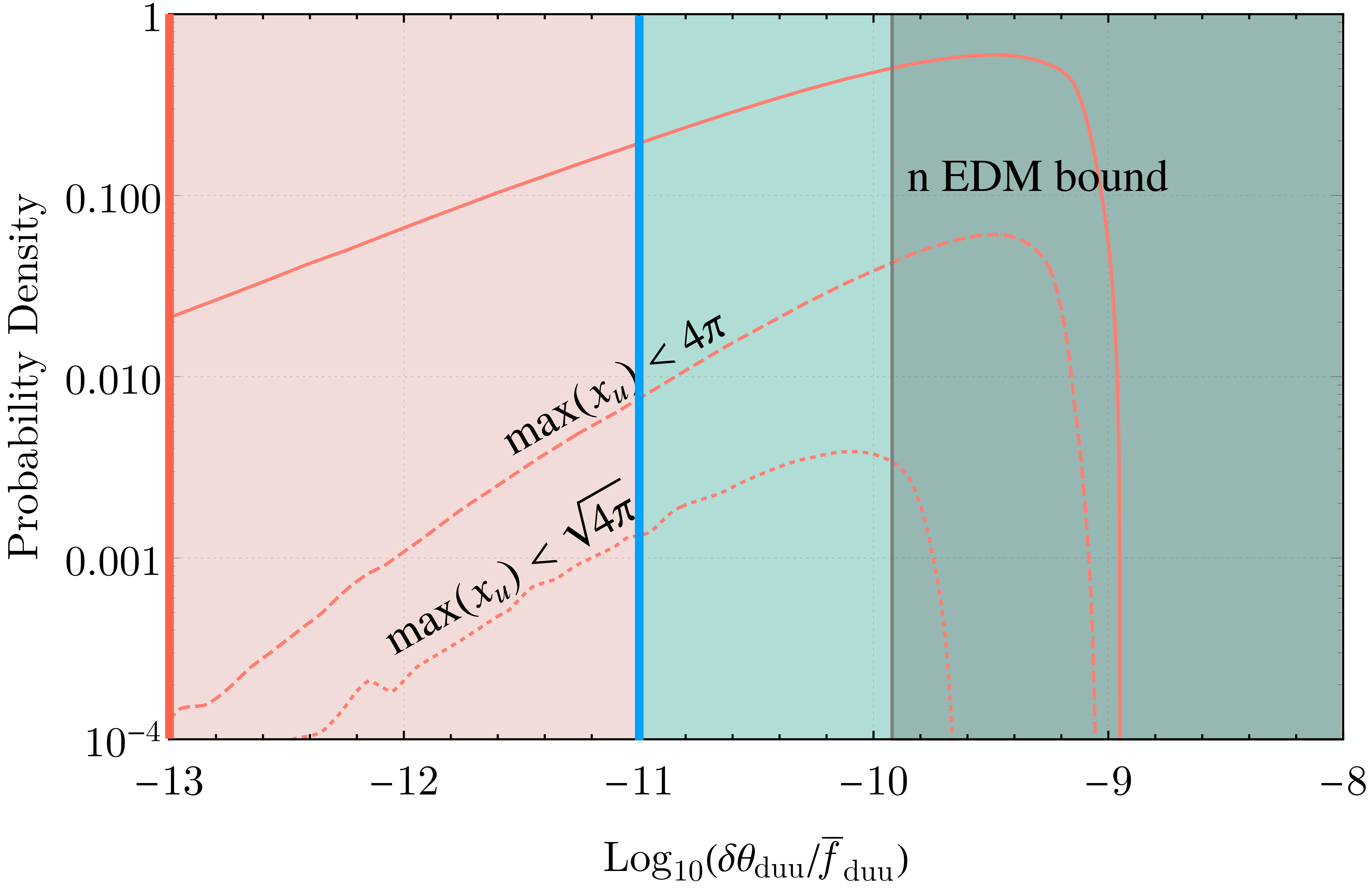
- CERN, CPEDM collaboration
- storage ring $$\frac{d\mathbf{s}}{dt} = \boldsymbol{\mu} \times \mathbf{B} + \mathbf{d} \times \mathbf{E} \quad (\mathbf{s}\text{: spin})$$
- $|d_p| \lesssim 10^{-29} e\,\mathrm{cm}$
- $\bar{\theta}$ parameter
  - improves at three order from nEDM




J. Alexander, *et al.*, arXiv:2205.00830 [hep-ph]

# PDF and future experiments




$$M_u^3/M_u^1 = 10^{-3}\,,\; M_u^1 = M_u^2$$

excluded region

**nEDM ($|d_n| \lesssim 1 \times 10^{-27} e\,\mathrm{cm}$)**

| | |
|---|---|
| $\max(x_u) < 4\pi$ | 94.0% |
| $\max(x_u) < \sqrt{4\pi}$ | 82.5% |

**pEDM ($|d_p| \lesssim 1 \times 10^{-29} e\,\mathrm{cm}$)**

| | |
|---|---|
| $\max(x_u) < 4\pi$ | 99.9% |
| $\max(x_u) < \sqrt{4\pi}$ | 99.8% |

# $\theta$ parameter respect to $SU(2)_L, SU(2)_R$

$$\mathcal{L} \ni \theta_2 \frac{\alpha_2}{8\pi} W^{\hat{a}}_{\mu\nu} \tilde{W}^{\hat{a}\mu\nu} + \theta_2 \frac{\alpha_2}{8\pi} W'^{\hat{a}}_{\mu\nu} \tilde{W}'^{\hat{a}\mu\nu}$$
$$+ \bar{Q}^i_L x^{ia}_u U^a_R \tilde{H} + \bar{Q}^i_R x^{ia}_u U^a_L \tilde{H}' + M^a_u \bar{U}^a_L U^a_R$$
$$+ \bar{Q}^i_L x^{ia}_d D^a_R H + \bar{Q}^i_R x^{ia}_d D^a_L H' + M^a_d \bar{D}^a_L D^a_R + \text{h.c.}$$

1: Two $\theta$ terms has a same coupling $\theta_2$ due to $P_{\text{gen}}$

2: axial $U(1)$ angle is $\beta = \frac{1}{2}(\theta_R - \theta_L)$

chiral rotation: $Q_L \to e^{i\theta_L} Q_L, \quad Q_R \to e^{i\theta_R} Q_R$

$SU(2)_L \qquad SU(2)_R$

3: $U, D$ have vector-like symmetry

**proceeding study**

sphaleron proces

$$\exp\left(-\frac{8\pi^2}{g_L^2} - \frac{8\pi^2}{g_R^2}\right) = 10^{-169}$$

$$g_L = g_R = 0.637$$

A. A. Anselm, A. A. Johansen, Nucl. Phys. *B* 412 (1994) 553-573

**The $\theta$ parameter of $SU(2)_L, SU(2)_R$ can be an observables, but it is too small to verify.**

# Neutrino

If the LR symmetry is generated to the lepton sector,

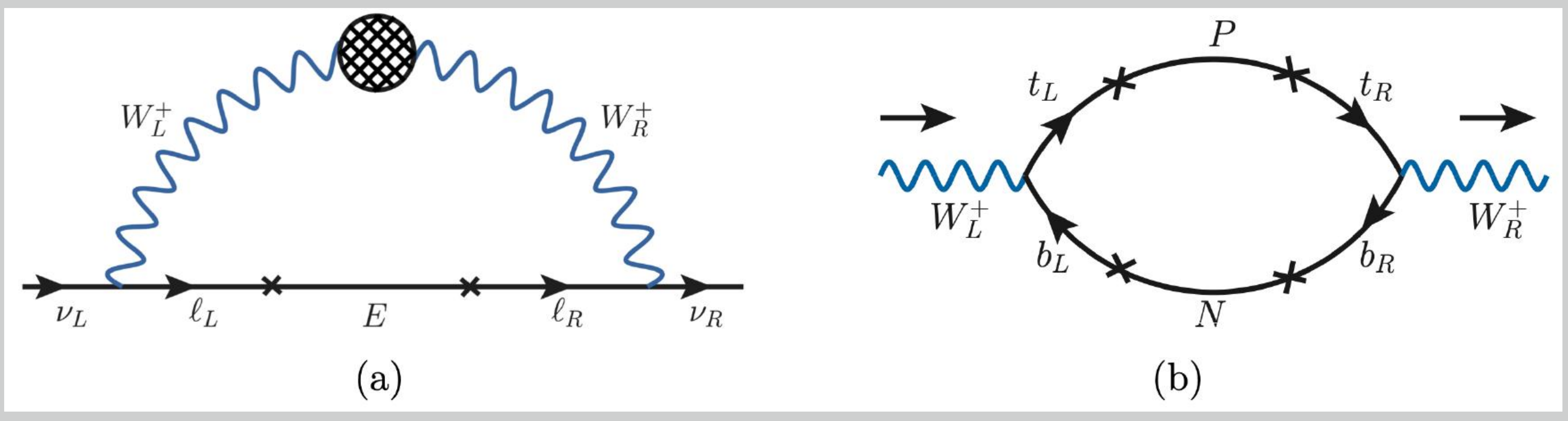
- no neutral VL lepton

$$\Psi_L(1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R(1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \quad E_{L/R}, \quad \cancel{N_{L/R}}$$

- charged lepton mass matrix

$$\mathcal{M}_e = \begin{pmatrix} 0 & x_e v \\ x_e^\dagger v' & M_E \end{pmatrix}$$

Dirac neutrino mass




**Certain benchmark points can explain the neutrino oscillation.**

| Oscillation parameters | $3\sigma$ range NuFit5.1 [48] | Model prediction | | | |
|---|---|---|---|---|---|
| | | BP I (NH) | BP II (NH) | BP III (IH) | BP IV (IH) |
| $\Delta m_{21}^2 (10^{-5}\ \mathrm{eV}^2)$ | 6.82 - 8.04 | 7.42 | 7.38 | 7.35 | 7.35 |
| $\Delta m_{23}^2 (10^{-3}\ \mathrm{eV}^2)$(IH) | 2.410 - 2.574 | - | - | 2.48 | 2.52 |
| $\Delta m_{31}^2 (10^{-3}\ \mathrm{eV}^2)$(NH) | 2.43 - 2.593 | 2.49 | 2.51 | - | - |
| $\sin^2\theta_{12}$ | 0.269 - 0.343 | 0.324 | 0.301 | 0.306 | 0.310 |
| $\sin^2\theta_{23}$ (IH) | 0.410 - 0.613 | - | - | 0.510 | 0.550 |
| $\sin^2\theta_{23}$ (NH) | 0.408 - 0.603 | 0.491 | 0.533 | - | - |
| $\sin^2\theta_{13}$ (IH) | 0.02055 - 0.02457 | - | - | 0.0219 | 0.0213 |
| $\sin^2\theta_{13}$(NH) | 0.02060 - 0.02435 | 0.0234 | 0.0213 | - | - |
| $\delta_{CP}$ (IH) | 192 - 361 | - | - | 236° | 279° |
| $\delta_{CP}$ (NH) | 105 - 405 | 199° | 280° | - | - |
| $m_{\mathrm{light}}$ $(10^{-3})$ eV | | 0.66 | 2.04 | 14.1 | 8.50 |
| $M_{E_1}/M_{W_R}$ | | 917 | 45.5 | 1936 | 1990 |
| $M_{E_2}/M_{W_R}$ | | 0.650 | 0.43 | 0.12 | 0.11 |
| $M_{E_3}/M_{W_R}$ | | 0.019 | 0.029 | 0.015 | 0.012 |

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