

# Axion, QCD angle, and the Weinberg operator for EDMs

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"Searches of EDMs: from Theory to Experiment"  
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Center for Theoretical Physics of the Universe



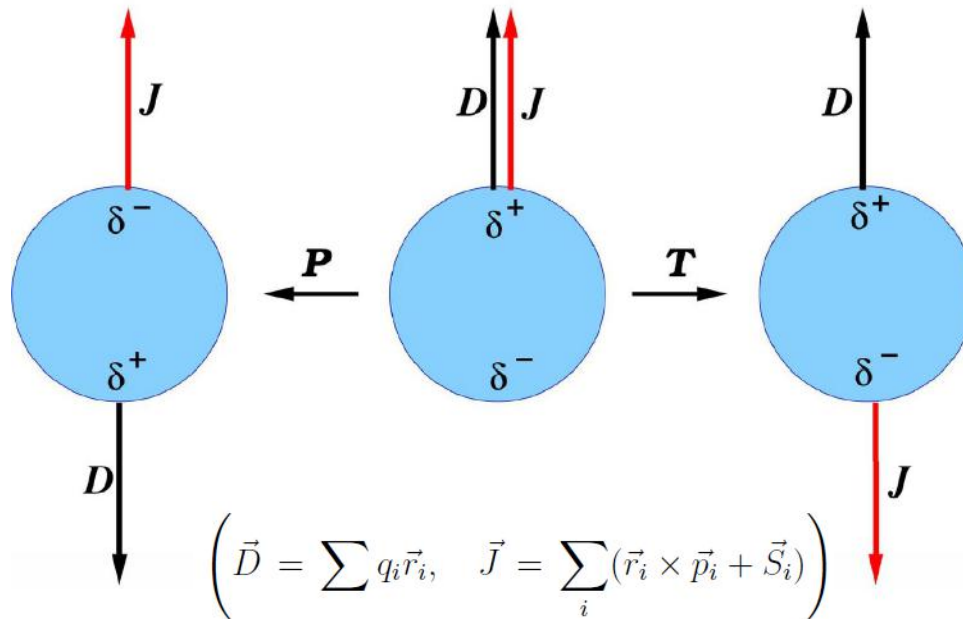
# Outline

- Introduction: EDM, axion and the PQ-quality problem
- PQ-quality problem for a QCD axion (=QCD angle in models with axion) in string theory
- Distinguishing the Weinberg operator (& CEDM) from the QCD angle by EDMs

## Why EDM is interesting and important?

Nonzero permanent EDM means **P** and **T (=CP)** violation.

Historically the violation of these discrete spacetime symmetries have played important role for the progress in fundamental physics.



CP violation is one of the key conditions to generate the asymmetry between matter and antimatter in our universe. Sakharov '67

Observed asymmetry:  $Y_B = \frac{n_B}{s} \sim 10^{-10}$

Standard Model (SM) prediction:  $(Y_B)_{\text{SM}} \lesssim 10^{-15}$

SM can provide neither an enough CP violation, nor out of equilibrium (hep-ph/0309291).

We need "new physics beyond the SM (BSM), involving CP violation", and EDM may provide a hint for those BSM physics.

EDM may also provide information useful for understanding the strong CP problem.

Two CP-violating angles in the SM (ignore neutrino masses):

\* Kobayashi-Maskawa phase:  $\delta_{\text{KM}} = \arg \cdot \text{Det}([y_u y_u^\dagger, y_d y_d^\dagger])$

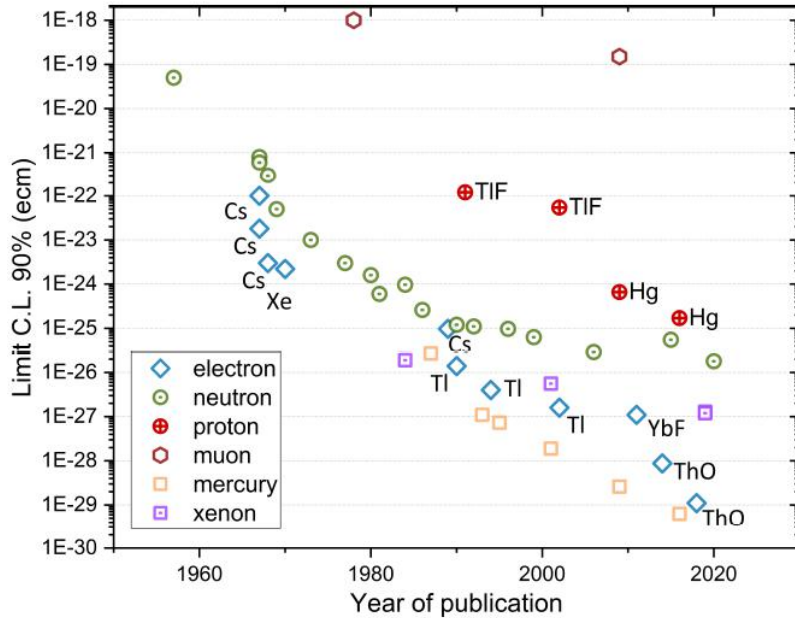
\* QCD angle:  $\bar{\theta} = \theta_{\text{QCD}} + \arg \cdot \text{Det}(y_u y_d)$

Experimental data imply

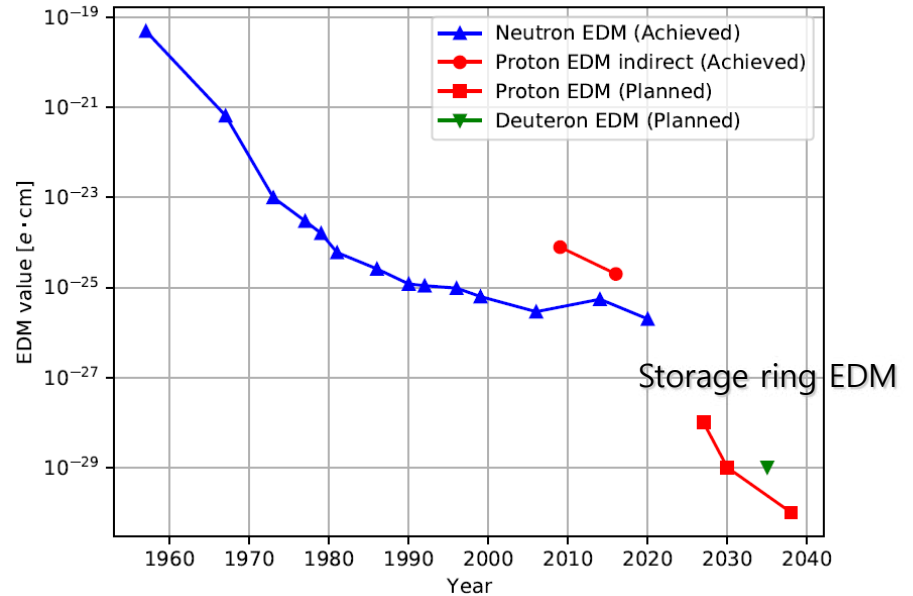
$$\delta_{\text{KM}} \sim 1, \quad |\bar{\theta}| \lesssim 10^{-10}$$

Strong CP problem: Why these two angles are so different?

EDMs have a bright prospect for experimental progress.



arXiv:2003.00717



arXiv:2203.08103

$$d_n < 1.8 \times 10^{-26} e \text{ cm}$$

$$d_e < 4.1 \times 10^{-30} e \text{ cm}$$

$$d_{Hg} < 7.4 \times 10^{-30} e \text{ cm}$$

Abel et al '20  
Roussy et al '22  
Graner et al '16

## SM predictions

$$\frac{d_n}{e \cdot \text{cm}} = -(1.5 \pm 0.7) \times 10^{-16} \sin \bar{\theta} + \mathcal{O}(10^{-31} - 10^{-32}) \times \sin \delta_{\text{KM}}$$

$$\frac{d_p}{e \cdot \text{cm}} = (1.1 \pm 1.0) \times 10^{-16} \sin \bar{\theta} + \mathcal{O}(10^{-31} - 10^{-32}) \times \sin \delta_{\text{KM}}$$

$$\frac{d_D}{e \cdot \text{cm}} = -(0.4 \pm 1.5) \times 10^{-16} \sin \bar{\theta} + \mathcal{O}(10^{-31} - 10^{-32}) \times \sin \delta_{\text{KM}}$$

De Vries et al '01

Mannel, Uraltsev '12

$$\rightarrow |\bar{\theta}| \lesssim 10^{-10}$$

$$\frac{d_e}{e \cdot \text{cm}} = -(2.2 - 8.6) \times 10^{-28} \sin \bar{\theta} + \mathcal{O}(10^{-44}) \times \sin \delta_{\text{KM}} \lesssim \mathcal{O}(10^{-37})$$

KC, Hong '91; Ghosh, Sato '18

Pospelov, Ritz '14

$$\frac{d_e^{\text{equiv}}}{e \cdot \text{cm}} \simeq 4.5 \times 10^{-22} \sin \bar{\theta} + 10^{-35} \sin \delta_{\text{KM}} \lesssim \mathcal{O}(10^{-31})$$

Flambaum et al '19

Ema et al '22

(due to  $\bar{e}\gamma_5 e \bar{N}N$  for paramagnetic molecules)

## EDMs in the SM

EDMs originating from  $\delta_{KM}$  are well below the experimental bounds.

On the other hand, hadronic EDMs from  $\bar{\theta}$  can have any value below the current experimental bounds.

Therefore, to identify the physical origin of experimentally observed hadronic EDMs (QCD angle vs new physics), quantitative understanding of the contribution from  $\bar{\theta}$  is essential, together with the measurements of multiple EDMs.



## $\bar{\theta}$ with axion

One may think we can eliminate  $\bar{\theta}$  by introducing a QCD axion which would solve the strong CP problem by the Peccei-Quinn (PQ) mechanism.

However, in modern viewpoint, even in theories with QCD axion,  $\bar{\theta}$  remains to be an incalculable free parameter, although its smallness becomes more natural (PQ-quality problem).

The axion solution to the strong CP problem is based on a global U(1) symmetry:

$$U(1)_{\text{PQ}} : a(x) \rightarrow a(x) + \text{constant}$$

which is **dominantly** broken by the axion coupling to gluons:

Peccei, Quinn; Weinberg; Wilczek

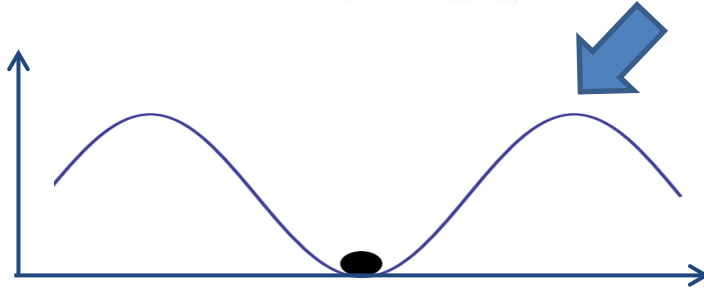
Effective lagrangian at  $E \sim 1 \text{ GeV}$ :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{32\pi^2} \frac{a}{f_a} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \Delta\mathcal{L}$$

Additional interactions

Additional axion potential

$$\Rightarrow V(a) = -\frac{m_\pi^2 f_\pi^2}{m_u + m_d} \left( m_u^2 + m_d^2 + 2m_u m_d \cos\left(\frac{a}{f_a}\right) \right)^{1/2} + \Delta V(a)$$



Shift of axion VEV

$$\Rightarrow \bar{\theta} \equiv \frac{\langle a \rangle}{f_a} = 0 + \dots$$

# Possible source of additional axion potential

CPV interactions of light quarks and gluons  
at  $E \sim 1$  GeV, including BSM physics contributions

Additional axion potential  
from PQ-breaking UV physics,  
e.g. quantum gravity

$$\Delta\mathcal{L} = \sum_i \lambda_i \mathcal{O}_i + \epsilon_{\text{QG}} \Lambda^4 \cos\left(\frac{a}{f_a} + \delta_{\text{QG}}\right)$$

$\{\mathcal{O}_i\} = \{(\bar{q}Dq)(\bar{q}q)(\bar{q}i\gamma_5q), \bar{q}\sigma \cdot G i\gamma_5q, GG\tilde{G}, \dots\}$

$$\bar{\theta} = \frac{\langle a \rangle}{f_a} = \sum_i \lambda_i \frac{\partial \bar{\theta}}{\partial \lambda_i} + \Delta\theta_{\text{QG}} \quad \left(\Delta\theta_{\text{QG}} \sim \frac{\epsilon_{\text{QG}} \Lambda^4}{m_\pi^2 f_\pi^2} \sin \delta_{\text{QG}}\right)$$

CP-violating observables including EDMs

$$D_I = \sum_i \lambda_i \frac{\partial D_I}{\partial \lambda_i} + \Delta\theta_{\text{QG}} \frac{\partial D_I}{\partial \bar{\theta}} \quad (I = n, p, D, Hg, Ra, \dots)$$

## PQ-quality problem:

How to protect  $U(1)_{\text{PQ}}$  from quantum gravity to make  $\Delta\theta_{\text{QG}}$  small enough?

In modern viewpoint, the presence of quantum gravity effects which break  $U(1)_{PQ}$  is considered to be an **unavoidable** feature:

(Swampland conjecture)

Black hole evaporation?, Gravitational Euclidean wormholes?,

String world-sheet or brane instantons?, ...

Barr et al '92

Kamionkowski et al '92

Holman et al '92

PQ-quality problem is important for EDM as it is about  $\bar{\theta}$  in theories with axion, so needs to be more concretely studied.

String theory is the right place to address the PQ-quality problem:

- (i) String theory is the best candidate for a theory of quantum gravity,
- (ii) String theory generically predicts axions.

String theory provides axions with a PQ symmetry which is locally equivalent to a gauge symmetry in the extra dimension.

### Similar simple example

5-dim gauge field  $A_M = (A_\mu, A_5)$  on  $M_4 \times S^1$  with  $x^5 \cong x^5 + 2\pi R$

5-dim gauge symmetry:

$$U(1)_{\text{gauge}} : A_5 \rightarrow A_5 + \frac{\partial \Lambda(x^\mu, x^5)}{\partial x^5}$$
$$(\Lambda(x^\mu, x^5 + 2\pi R) = \Lambda(x^\mu, x^5))$$

4-dim axion from 5-dim gauge field:

$$a(x^\mu) = \frac{1}{2\pi R} \oint dx^5 A_5(x^\mu, x^5)$$

Associated PQ-symmetry is locally equivalent to 5-dim gauge symmetry:

$$U(1)_{\text{PQ}} : a(x^\mu) \rightarrow a(x^\mu) + \beta \quad (\beta = \text{constant})$$

$$a(x^\mu) = \frac{1}{2\pi R} \oint dx^5 A_5(x^\mu, x^5)$$

$$U(1)_{\text{gauge}} : A_5 \rightarrow A_5 + \frac{\partial \Lambda(x^\mu, x^5)}{\partial x^5}$$

$$(\Lambda(x^\mu, x^5 + 2\pi R) = \Lambda(x^\mu, x^5))$$

$U(1)_{\text{PQ}} = U(1)_{\text{gauge}}$  for  $\Lambda = \beta x^5$  which is only locally well-defined on  $S^1$

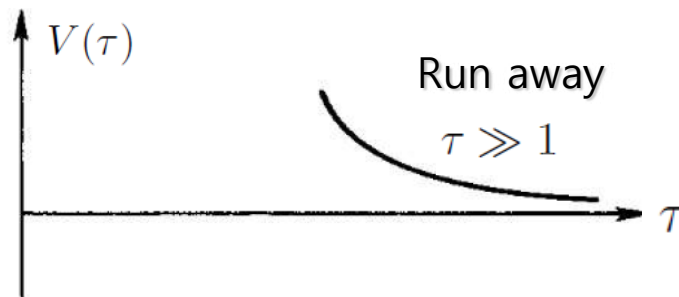
➔  $U(1)_{\text{PQ}}$  is broken only by non-local effects which are exponentially suppressed by  $\exp(-2\pi R)$ .

Similarly, in string theory

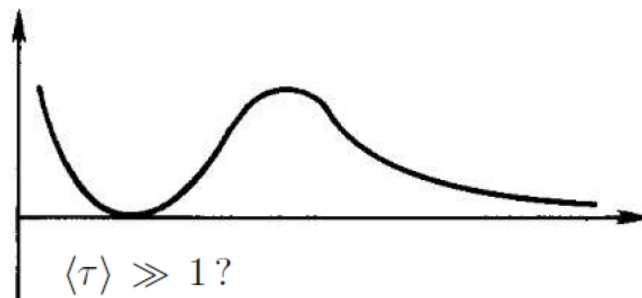
$$\Delta\theta_{QG} \propto \exp(-2\pi\tau) \quad (\tau = \text{scalar partner of the pseudo-scalar axion})$$

so  $\Delta\theta_{QG}$  can be small enough if  $\tau$  has a large enough vacuum value.

On the other hand, one can not simply assume  $\tau \gg 1$  because



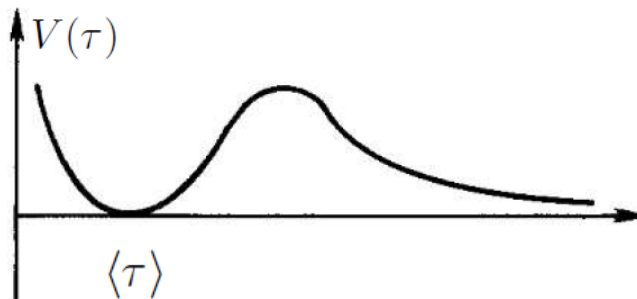
So the right question is "Can the stabilized value of  $\langle\tau\rangle$  be large enough to give  $|\Delta\theta_{QG}| < 10^{-10}$  ?



String theory provides a large set of models called a string landscape, rather than selecting a specific model.

Then a more suitable question is "What would be the distribution of  $\Delta\theta_{\text{QG}}$  over the landscape of models involving a QCD axion (=approximate PQ-symmetry), as well as a dynamics to fix the VEV of its scalar partner?".

This is equivalent to the question about the distribution of  $\langle\tau\rangle$  over the corresponding landscape:

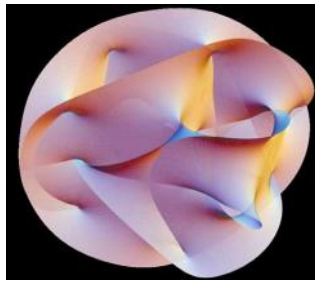




Models in string theory with QCD axion whose scalar partner VEV is fixed a la (generalized) KKLT moduli stabilization

Kachru, Kallosh, Linde, Trivedi, '03

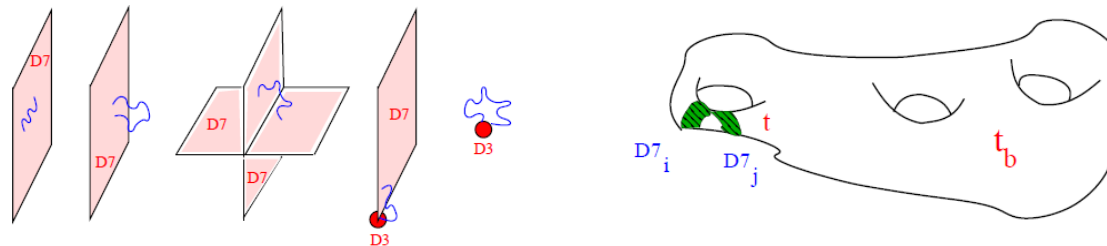
Compactified 6-dim internal space involving a variety of closed p-dim surfaces:



$h_{1,1}$  surfaces with intersection numbers

$$d_{ijk} \quad (i, j, k = 1, 2, \dots, h_{1,1})$$

Branes supporting the hidden gauge fields while wrapping the closed surfaces in the internal space:



$N_h$  sets of  $C_h$  stacks of branes wrapping the  $i$ -th surface  $\ell_h^i$ -times

$$(h = 1, \dots, N_h; i = 1, \dots, h_{1,1} > N_h)$$

(to have a QCD axion)

Exponentially large set of models described by many independent integer-valued parameters describing the topological structures of the 6-dim internal space and the possible brane configurations:

$$h_{1,1}, d_{ijk}, N_h, C_h, \ell_h^i$$

Nearly flat distribution of  $\log |\Delta\theta_{\text{QG}}|$  over the landscape:

KC, Im, Jeong, Yun, in preparation

$$\log |\Delta\theta_{\text{QG}}| \simeq -5.1 + \log \left( \frac{m_{3/2}}{10 \text{ TeV}} \right) + 0.43 \times \left( 157 - \frac{1}{2} \sum_{jk} \sum_{hh'} \xi_h \xi_{h'} d_{ijk} \ell_h^j \ell_{h'}^k \right)$$

$$\sum_{h'h''} \left( \sum_{ijk} d_{ijk} \ell_h^i \ell_{h'}^j \ell_{h''}^k \right) \xi_{h'} \xi_{h''} = 2C_h \ln(M_{\text{Pl}}/m_{3/2}) \quad (h = 1, \dots, N_h)$$



\* It is not implausible that  $\log |\Delta\theta_{\text{QG}}| < -10$ .

\*  $\log |\Delta\theta_{\text{QG}}|$  can have any value  $< -10$  with a nearly equal probability.

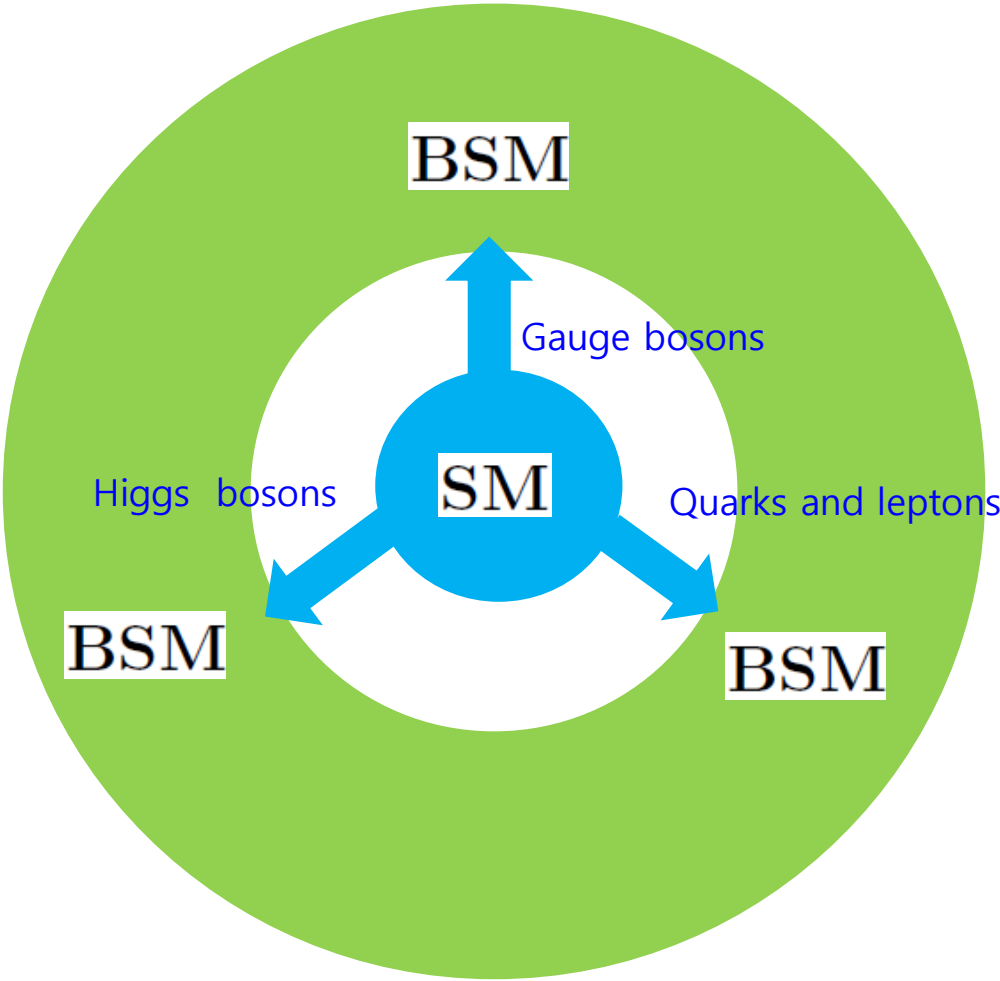
So regardless of the existence of axion, it is important to examine if EDMs from any BSM physics are distinguishable from those due to the QCD angle.

This issue has been studied recently, but only for the models in which BSM physics communicate with the SM mainly through the SM quarks and leptons.

(Leptoquarks; LR-symmetric; MSSM in certain parameter region)

De Vries et al '21

BSM physics might be classified by how it communicates with the SM.



There are BSM physics which communicates with the SM mainly through the gauge bosons or the Higgs boson, while being relatively sequestered from the SM quark and leptons:

Certain type of DM, vector-like exotic matter,  
hidden confining sector, ...

In such models, EDM often provides the most sensitive tool to probe BSM physics.

# Effective theory approach for EDM from BSM physics

BSM model at  $E > 1 \text{ TeV}$

Integrate out the massive  
BSM particles



CPV operators  
of the SM fields

$$f^{abc} G^a G^b \tilde{G}^c + |H|^2 G \tilde{G} + H \bar{Q}_L \sigma^{\mu\nu} G_{\mu\nu} d_R \\ + H \bar{Q}_L \sigma^{\mu\nu} B_{\mu\nu} d_R + \bar{L}_L e_R \bar{d}_R Q_L + \dots$$

Integrate out the massive  
SM particles



CPV operators of  
the light fields at  
 $E \sim 1 \text{ GeV}$

Weinberg operator

Quark chromo-EDM (CEDM)

$$f^{abc} G^a G^b \tilde{G}^c + \bar{q} \sigma^{\mu\nu} i \gamma_5 G_{\mu\nu} q \\ \bar{q} \sigma^{\mu\nu} i \gamma_5 F_{\mu\nu} q + \bar{e} \sigma^{\mu\nu} i \gamma_5 F_{\mu\nu} e + \dots$$

Quark and electron EDM

4-fermion operators

Hadronic & atomic physics



EDMs

For BSM physics which communicates with the SM mainly through the gauge bosons and/or the Higgs boson, the Weinberg operator and quark CEDM play major role for hadronic EDMs.

Simple example: Vector-like quarks

KC, Kim, Im, Mo ,16

$$m_\psi \psi \psi^c + y_s S \psi \psi^c + \text{h.c.}$$

$\psi + \psi^c$  : vector-like quark

$S$  : singlet real scalar



$m_\psi, y_s$  : complex

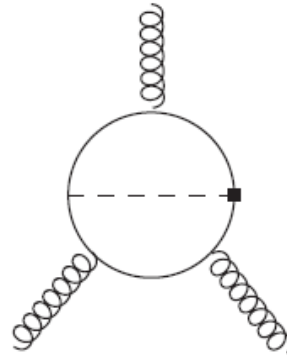
$$|m_\psi| \bar{\Psi} \Psi + |y_s| \cos \alpha S \bar{\Psi} \Psi + i |y_s| \sin \alpha S \bar{\Psi} \gamma^5 \Psi$$

$$\Psi = \begin{pmatrix} \psi \\ \psi^{c*} \end{pmatrix}$$

$$\alpha = \arg(m_\psi) - \arg(y_s)$$

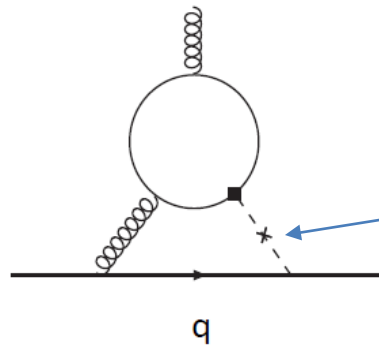
# Leading CPV from Vector-like quarks

Weinberg operator



$$\frac{1}{3!} d_W f^{abc} \epsilon^{\mu\nu\rho\sigma} G_{\lambda\mu}^a G_{\nu\rho}^b G_{\sigma}^{c\lambda}$$

quark CEDM



$$\frac{1}{2} \tilde{d}_q g_s \bar{q} \sigma^{\mu\nu} i \gamma^5 q G_{\mu\nu}$$

s-h mixing

Relative importance of CEDM depends on the size of s-h mixing.



## Subsequent RGE at 1-loop

$$C_1(\mu) = \frac{d_q(\mu)}{m_q Q_q}, \quad C_2(\mu) = \frac{\tilde{d}_q(\mu)}{m_q}, \quad C_3(\mu) = \frac{d_W(\mu)}{g_3}$$

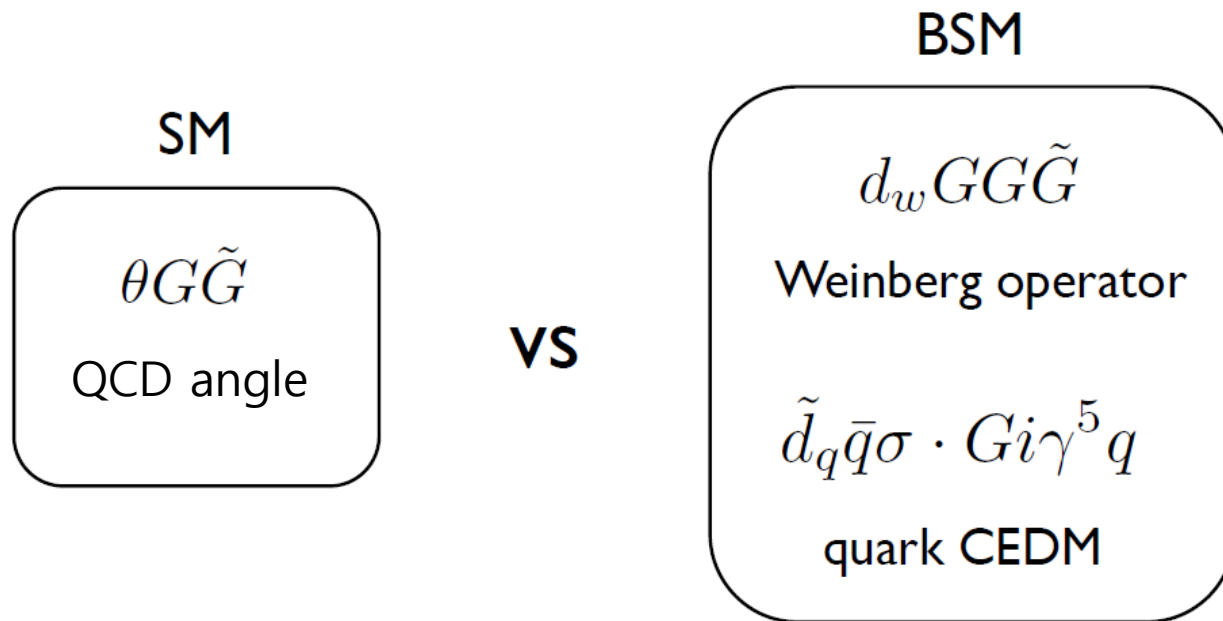
$$\mu \frac{\partial \mathbf{C}}{\partial \mu} = \frac{g_3^2}{16\pi^2} \gamma \mathbf{C}$$

$$\gamma \equiv \begin{pmatrix} \gamma_e & \gamma_{eq} & 0 \\ 0 & \gamma_q & \gamma_{Gq} \\ 0 & 0 & \gamma_G \end{pmatrix} = \begin{pmatrix} 8C_F & 8C_F & 0 \\ 0 & 16C_F - 4N_c & 2N_c \\ 0 & 0 & N_c + 2n_f + \beta_0 \end{pmatrix}$$

- $C_F = \frac{4}{3}$  (quadratic Casimir)
- $N_c = 3$
- $n_f =$  number of light quarks
- $\beta_0 = (33 - 2n_f)/3$

# Distinguishability from the QCD angle

KC, Im, Jodlowski, in preparation



Do they predict different CPV signals?

# Nucleon EDMs

NDA

$$d_N = \frac{e}{\Lambda_\chi} \left( \pm \frac{m_*}{\Lambda_\chi} \bar{\theta} \pm \frac{\Lambda_\chi^2}{4\pi} d_w \pm \frac{\Lambda_\chi}{4\pi} \tilde{d}_q \right) \quad \Lambda_\chi = 4\pi J_\pi$$

$$m_* \equiv (\text{tr} M_q^{-1})^{-1} \simeq \frac{m_u m_d}{m_u + m_d}$$

Agrees  
more  
or less



QCD  
sum rules

$$d_p = -1.2 \times 10^{-16} [e \text{ cm}] \bar{\theta} + e(20 \text{ MeV}) d_w$$

$$+ e(-0.28 \tilde{d}_u + 0.28 \tilde{d}_d + 0.021 \tilde{d}_s)$$

$$+ e(2.7 \text{ MeV})/\Lambda^2 \quad \tilde{d}_q \equiv m_q/\Lambda^2$$

$$d_n = 0.82 \times 10^{-16} [e \text{ cm}] \bar{\theta} - e(18 \text{ MeV}) d_w$$

$$+ e(-0.30 \tilde{d}_u + 0.30 \tilde{d}_d - 0.014 \tilde{d}_s)$$

$$- e(0.6 \text{ MeV})/\Lambda^2$$

Pospelov, Ritz '99  
Hisano, Lee, Nagata,  
Shimizu '12  
Hisano, Kobayashi,  
Kuramoto, Kuwahara '15  
Yamanaka, Hiyama '20



$d_p \approx -d_n$  except the CEDM contributions?  
(because of the isospin-invariance of CPV?)

With a QCD axion,

Pospelov and Ritz '00

$$\bar{\theta} = \frac{\langle a \rangle}{f_a} \simeq \frac{0.8 \text{ GeV}^2}{2} \sum_q \frac{\tilde{d}_q}{m_q} + \Delta\theta_{\text{QG}}$$

$$d_p^{PQ} = e(20 \text{ MeV})d_w + \underbrace{e(-1.2 \tilde{d}_u - 0.15 \tilde{d}_d)}_{-e(3.5 \text{ MeV})/\Lambda^2} - 1.2 \times 10^{-16} [e \text{ cm}] \Delta\theta_{\text{QG}}$$

Hisano, Kobayashi,

Kuramoto, Kuwahara '15

$$-e(3.5 \text{ MeV})/\Lambda^2 \quad \tilde{d}_q \equiv m_q/\Lambda^2$$

$$d_n^{PQ} = -e(18 \text{ MeV})d_w + \underbrace{e(0.29 \tilde{d}_u + 0.59 \tilde{d}_d)}_{+e(3.5 \text{ MeV})/\Lambda^2} + 0.82 \times 10^{-16} [e \text{ cm}] \Delta\theta_{\text{QG}}$$

All three sources of CPV give similar pattern of nucleon EDM:

$$d_p \approx -d_n$$

## Aids from (isospin-breaking) CP odd nuclear force

$$\bar{g}_0 \bar{N} \tau \cdot \pi N + \bar{g}_1 \bar{N} \pi_3 N$$

Isospin-singlet                      Isospin-breaking

Diamagnetic atomic EDMs are sensitive to the isospin-breaking CP-odd pion-nucleon coupling.

$$d_{Ra} = (7.7 \times 10^{-4}) \times [(2.5 \pm 7.5)\bar{g}_0 - (65 \pm 40)\bar{g}_1] e \text{ fm}$$

$$d_D = (0.94 \pm 0.01)(d_n + d_p) + [(0.18 \pm 0.02)\bar{g}_1] e \text{ fm}$$

de Vries, Draper, Fuyuto, Kozaczuk, Lillard '21

Osamura, Gubler, Yamanaka '22


- $d_{Ra} < 1.4 \times 10^{-23}$  e cm (to be improved up to  $1 \times 10^{-28}$  e cm) M Bishof et al '16
- $d_D$  has no limit yet but to be probed up to  $10^{-29}$  e cm in a storage ring experiment  
F. Abusaif et al. (CPEDM Collaboration) '19                      Yannis Semertzidis's talk

$$\boxed{\bar{g}_1 \bar{N} \pi_3 N}$$

NDA


$$\bar{g}_1 = \pm 4\pi \frac{(m_u - m_d)m_*}{\Lambda_\chi^2} \bar{\theta} \pm (m_u - m_d)\Lambda_\chi d_w \pm \Lambda_\chi (\tilde{d}_u - \tilde{d}_d)$$

ChPT & QCD sum rules;  
 agrees with NDA  
 (Osamura, Gubler, Yamanaka '22)




$$\bar{g}_1 = (3.4 \pm 2.4) \times 10^{-3} \bar{\theta} \pm (2.2 \pm 1.6) \times 10^{-3} \text{GeV}^2 d_w$$

ChPT & baryon spectrum;  
 $\gtrsim 4\pi \times \text{NDA}$   
 (de Vries, Mereghetti,  
 Walker-Loud '15)



$$\underbrace{+(28 \pm 12) \text{GeV} (\tilde{d}_u - \tilde{d}_d)}_{\tilde{d}_q \equiv m_q / \Lambda^2}$$

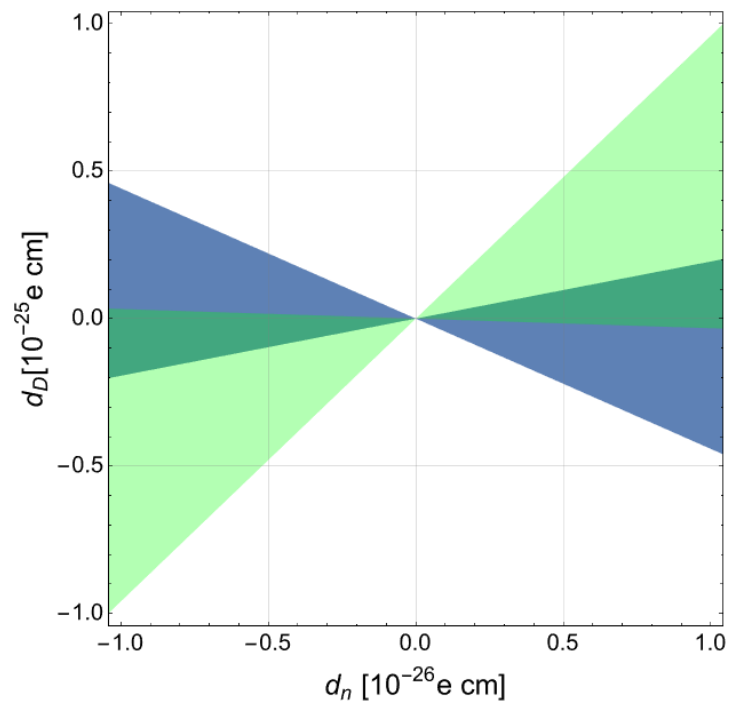
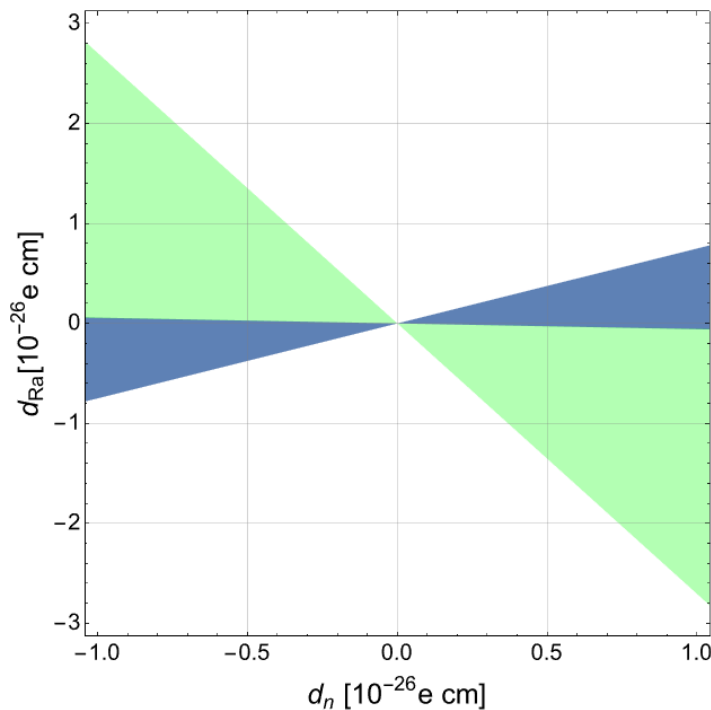
$$-(0.7 \pm 0.3) \times 10^{-1} \text{GeV}^2 / \Lambda^2$$


ChPT & QCD sum rules;  
 enhanced by  $\frac{\sigma_{\pi N}}{\bar{m}} \sim 4\pi$   
 (de Vries et al '21)

Can we distinguish the Weinberg-operator-dominated CPV at  $\Lambda_{\text{BSM}} \geq 1 \text{ TeV}$  from the QCD angle?

KC, Im, Jodlowski, in preparation

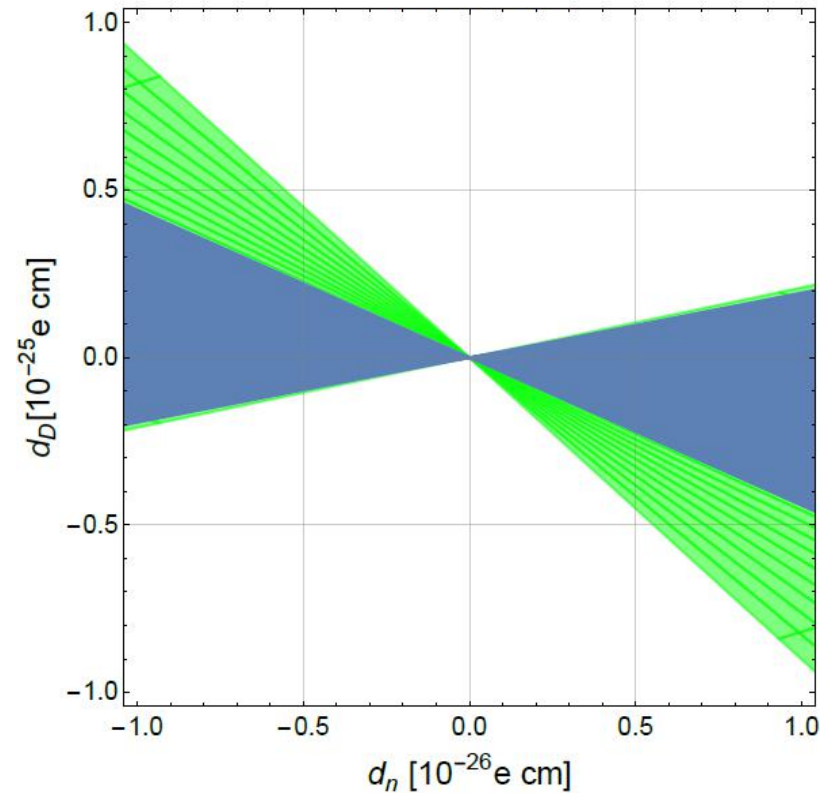
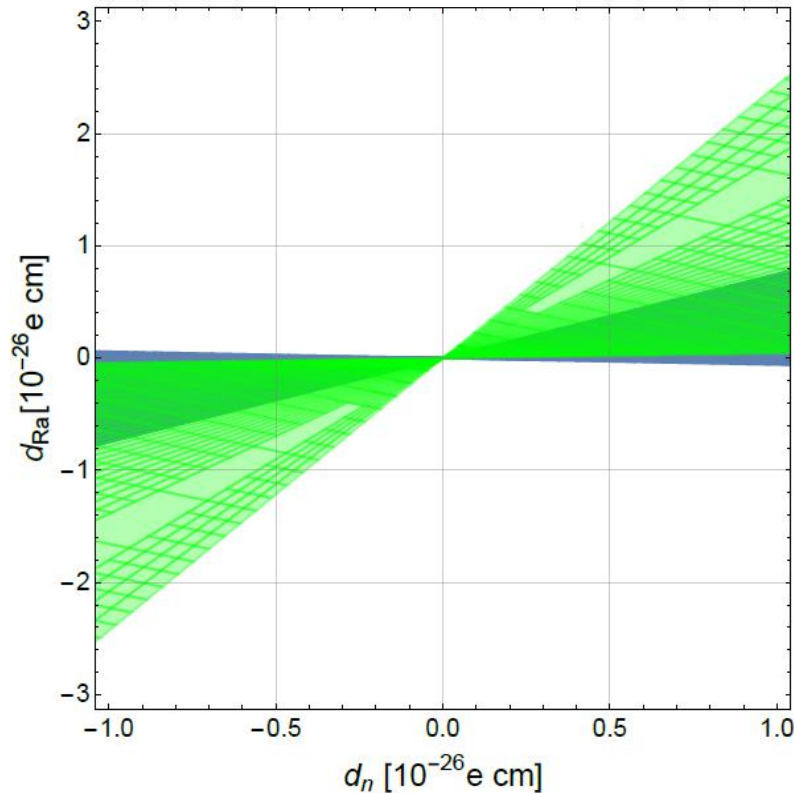
Relative sign (I) (RG-induced CEDMs included)



Blue: QCD angle

Green: Weinberg operator at  $\Lambda_{\text{BSM}} = 1 \text{ TeV}$

## Relative sign (II) (RG-induced CEDMs included)



Further extensive studies of the hadronic and atomic physics parts are required to distinguish the Weinberg operator & CEDM contributions from the QCD angle contribution.



# Conclusion

- EDM is interesting and important.
  - \* Matter-antimatter asymmetry, Strong CP problem, .....
  - \* For BSM physics which communicate to the SM mainly through the gauge bosons and/or the Higgs boson, EDM may provide the most sensitive tool to probe new physics.
- Even with axion, the QCD angle may have any value below the current bound  $\sim 10^{-10}$ .
- Distinguishing BSM physics contributions from the QCD angle contributions to hadronic EDMs appears to be quite challenging, and it requires an extensive study of the involved hadronic and atomic physics.