Contributions of CP Violating Operators to the Neutron/Proton EDM from Lattice QCD:

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LANL EDM collaboration

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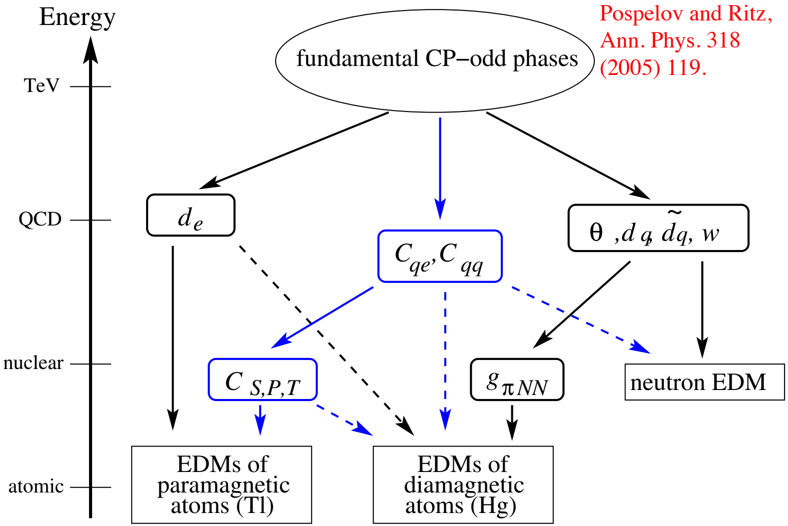
LANL Publications

- Bhattacharya et al, “Dimension-5 CP-odd operators: QCD mixing and renormalization”, PhysRevD.92.114026
- Bhattacharya et al, “Neutron Electric Dipole Moment and Tensor Charges from Lattice QCD”, PhysRevLett.115.212002
- Bhattacharya et al, “Isovector and isoscalar tensor charges of the nucleon from lattice QCD”, PhysRevD.92.094511
- Gupta et al, “Flavor diagonal tensor charges of the nucleon from (2 + 1 + 1)-flavor lattice QCD” PhysRevD.98.091501
- Bhattacharya et al, “Neutron Electric Dipole Moment from Beyond the Standard Model”, arXiv:1812.06233, PoS LATTICE2018 (2018) 188.
- Bhattacharya et al, “Contribution of the QCD Θ-term to nucleon electric dipole moment”, PhysRevD.103.114507.
- Bhattacharya et al, ”nEDM from the theta-term and chromoEDM operators”, arXiv:2301.08161, PoS LATTICE2022 (2023) 304

Hierarchy of Scales: Effective Field Theory



Pospelov and Ritz, Ann. Phys. 318 (2005) 119.

$$\mathcal{L}_{\rm CPV} = \mathcal{L}_{\rm CKM} + \mathcal{L}_{\bar\theta} + \mathcal{L}_{\rm BSM} \quad \longrightarrow \quad \mathcal{L}^{\rm eff}_{\rm CPV}$$

Effective CPV Lagrangian at Hadronic Scale

$$
\begin{aligned}
\mathcal{L}^{d\leq 6}_{\mathrm{CPV}} &= -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
&- \frac{i}{2}\sum_{q=u,d,s,c} d_q\overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
&- \frac{i}{2}\sum_{q=u,d,s,c} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
&+ d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 3g Weinberg operator} \\
&+ \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

- $\overline{\theta} \leq \mathcal{O}(10^{-9} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \langle \mathrm{v}\rangle/\Lambda^2_{BSM}$; effectively dim=6
- Dim=6 terms suppressed by $d_w \approx 1/\Lambda^2_{BSM}$
- All terms up to $d = 6$ are leading order

Contributions to the Neutron EDM d_n

$$d_n = \overline{\theta} \cdot C_\theta + d_q \cdot C_{\mathrm{qEDM}} + \tilde{d}_q \cdot C_{\mathrm{qcEDM}} + \tilde{d}_w \cdot C_{\mathrm{W}} + \cdots$$

- SM and BSM theories
  $\longrightarrow$ Coefficients of the effective CPV Lagrangian ($\overline{\theta}, d_q, \tilde{d}_q, \ldots$)

- Lattice QCD
  $\longrightarrow$ Nucleon matrix elements in presence of CPV interactions

$$\begin{aligned}
C_\theta &= \langle N|J^{\mathrm{EM}}|N\rangle|_\theta \\
C_{\mathrm{qEDM}} &= \langle N|J^{\mathrm{EM}}|N\rangle|_{\mathrm{qEDM}} \\
C_{\mathrm{CEDM}} &= \langle N|J^{\mathrm{EM}}|N\rangle|_{\mathrm{CEDM}} \\
C_{\mathrm{W}} &= \langle N|J^{\mathrm{EM}}|N\rangle|_{\mathrm{Weinberg}} \\
&\ldots
\end{aligned}$$

Lattice QCD $\Longrightarrow$ Physical Results

- **Removing Excited state contamination**
  – Lattice meson and nucleon interpolating operators also couple to excited states
- **Renormalization: Lattice scheme $\longrightarrow$ continuum $\overline{\mathrm{MS}}$**
  – involves complicated/divergent mixing for C_{qcEDM} and C_{W}
- **Heavier $\rightarrow$ Physical Pion Mass.**
  – As $M_\pi \rightarrow 135$ MeV $\Longrightarrow$ larger errors as computational cost increases
- **Finite Lattice Spacing**
  – Extrapolate from finite lattice spacing $0.045 < a < 0.15$ fm
- **Finite Volume**
  – Finite lattice volume effects small in most EDM calculations for $M_\pi L > 4$

Extrapolate data at $\{a, M_\pi, M_\pi L\}$ to $a = 0$, $M_\pi = 135$ MeV, $M_\pi L \rightarrow \infty$

Neutron EDM from Quark EDM term

$$
\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

Contribution of the quark EDMs, C_{qEDM}

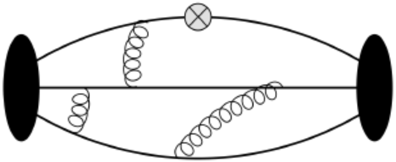
- On the addition of quarkEDM operator
  $\mathcal{L}_{\mathrm{CPV}} = \mathcal{L}_{\mathrm{CKM}} + \mathcal{L}_{\mathrm{qEDM}}$
- The electric current acquires an additional CPV piece, the tensor bilinear whose matrix elements are the tensor charges g_T
  $\langle N|\overline{q}\sigma_{\mu\nu}q|N\rangle = g_T^q \overline{u}_N \sigma_{\mu\nu} u_N$
- These tensor charges give the leading contributions of Quark EDMs

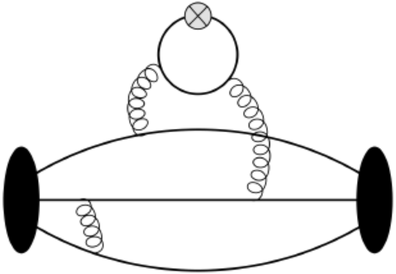
$$-\frac{i}{2}\sum_{q=u,d,s,c} d_q \overline{q}(\sigma\cdot F)\gamma_5 q \quad \longrightarrow \quad d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + d_c g_T^c$$

- $d_q \propto m_q$ in many models $\Rightarrow$ Precision determination of $g_T^{\{s,c\}}$ is important

Calculating the Tensor Charges

Need to calculate both the ”connected” and “Disconnected” diagrams





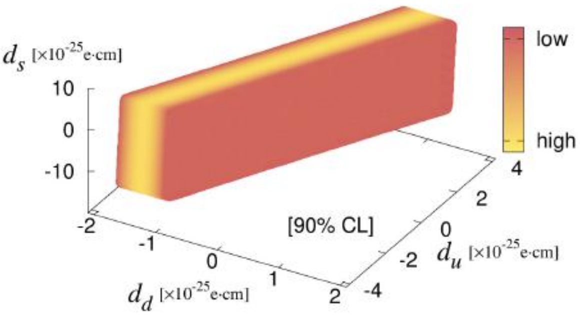
- ONLY the disconnected diagram contributes to $g_T^{\{s,c\}}$
- “Disconnected” diagrams are noisy (expensive) but fortunately small.
- Robust results to within 5% accuracy have been obtained

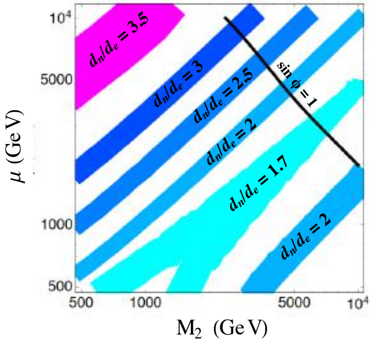
qEDM: FLAG2019, 2021 and Current Status

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^u	g_T^d
PNDME 20	2+1+1	★‡	★	★	★	○	0.783(27)(10)	−0.205(10)(10)
ETM 19	2+1+1	■	○	★	★	○	0.729(22)	−0.2075(75)
PNDME 18B	2+1+1	★‡	★	★	★	○	0.784(28)(10)#	−0.204(11)(10)#
PNDME 16	2+1+1	○‡	★	★	★	○	0.792(42)#&	−0.194(14)#&
Mainz 19	2+1	★	○	★	★	○	0.77(4)(6)	−0.19(4)(6)
JLQCD 18	2+1	■	○	○	★	○	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	2	■	○	○	★	○	0.782(16)(2)(13)	−0.219(10)(2)(13)

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^s
PNDME 20	2+1+1	★‡	★	★	★	○	−0.0022(12)
ETM 19	2+1+1	■	○	★	★	○	−0.00268(58)
PNDME 18B	2+1+1	★‡	★	★	★	○	−0.0027(16)#
Mainz 19	2+1	★	○	★	★	○	−0.0026(73)(42)
JLQCD 18	2+1	■	○	○	★	○	−0.012(16)(8)
ETM 17	2	■	○	○	★	○	−0.00319(69)(2)(22)

Constraints on BSM from qEDM and Future Prospects





[Bhattacharya, *et al.* (2015), Gupta, *et al.* (2018)]

Status:

- $g_T^{u,d,s}$ results from multiple collaborations with control over $a \to 0$ extrapolation
- Single result from ETM 19 $g_T^c = -0.00024(16)$

Neutron EDM from QCD θ-term

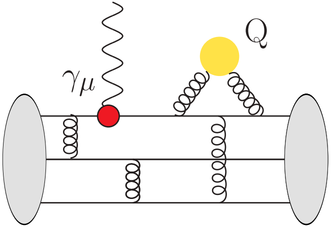
$$
\begin{aligned}
\mathcal{L}_{\rm CPV}^{d\leq 6} &= -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
&- \frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
&- \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
&+ d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
&+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

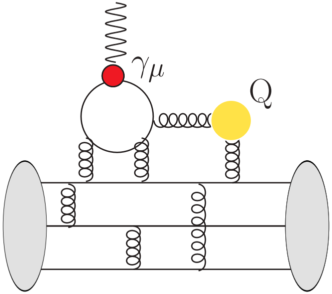
QCD θ-term

$$S = S_{QCD} + i\theta Q, \qquad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

At the leading order, the correlation functions calculated are

$$\langle N \mid J_\mu^{\rm EM} \mid N \rangle \Big|^{\overline{\Theta}} \approx \langle N \mid J_\mu^{\rm EM} \mid N \rangle \Big|^{\overline{\Theta}=0} - i\overline{\Theta} \left\langle N \middle| J_\mu^{\rm EM} \int d^4x \, \frac{G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a}{32\pi^2} \middle| N \right\rangle ,$$





Three different approaches for the QCD θ-term

- External electric field method: $\langle N\overline{N}\rangle_\theta(\vec{\mathcal{E}},t) = \langle N(t)\overline{N}(0)e^{i\theta Q}\rangle_{\vec{\mathcal{E}}}$

  Aoki and Gocksch (1989), Aoki, Gocksch, Manohar, and Sharpe (1990), CP-PACS Collaboration (2006), Abramczyk, *et al.* (2017)

- Simulation with imaginary θ: $\theta = i\tilde{\theta}$, $S^q_\theta = \tilde{\theta}\frac{m_l m_s}{2m_s+m_l}\sum_x \overline{q}\gamma_5 q$

  Horsley, *et al.*, (2008), Guo, *et al.* (2015)

- Expansion in small θ:

$$\langle O(x)\rangle_\theta = \frac{1}{Z_\theta}\int d[U,q,\overline{q}]O(x)e^{-S_{QCD}-i\theta Q}$$
$$= \langle O(x)\rangle_{\theta=0} - i\theta\langle O(x)Q\rangle_{\theta=0} + \mathcal{O}(\theta^2)$$

  Shintani, *et al.*, (2005); Berruto, *et al.*, (2006); Shindler *et al.* (2015); Shintani, *et al.* (2016); Alexandrou *et al.*, (2016) Abramczyk, *et al.* (2017) Dragos, *et al.* (2019); Alexandrou, *et al.* (2020); Bhattacharya, *et al.* (2021)

d_n from the form factor $F_3(0) = 2M_N d_n/\epsilon$

In the expansion in small 'couplin g' method, $F_3(0)$ is obtained from the most general decomposition of the matrix element:

$$
\begin{aligned}
\langle N(p',s') \mid J_\mu^{\mathrm{EM}} \mid N(p,s)\rangle^{\overline{\Theta}}_{\not{C}\mathrm{P}} \;=\; \overline{u}_N(p',s')\Bigg[\gamma_\mu F_1(q^2) \\
+ \frac{1}{2M_N}\sigma_{\mu\nu}q_\nu\Big(F_2(q^2) - iF_3(q^2)\gamma_5\Big) \\
+ \frac{F_A(q^2)}{M_N^2}(\not{q}q_\mu - q^2\gamma_\mu)\gamma_5\Bigg]u_N(p,s)\,,
\end{aligned}
$$

- Resolve the four form factors F_1, F_2, F_3, and F_A

3 steps in calculation of F_3 using expansion in small coupling method

- Determine the spinor phase α from nucleon 2-point function:
  - In a theory with P violation, the neutron state that satisfies the standard Dirac equation acquires a phase $e^{i\gamma_5\alpha}$. If correlation functions have been calculated without including α, then F_3 is not the correct CP-odd form factor because F_2 and F_3 mix.
  - There is a unique α for each:
    (i) Nucleon interpolating operator N;
    (ii) State created by N;
    (iii) Type of CPV interaction

- Remove excited state contributions from correlation functions to get ground state matrix element

- Extract F_3 from the ground-state matrix element

Spinor phase α with P and CP violation and impact on F_3

The most general spectral decomposition of the 2-point nucleon correlator is

$$\langle \Omega | \mathcal{T} N(\boldsymbol{p}, \tau) \overline{N}(\boldsymbol{p}, 0) | \Omega \rangle = \sum_{i, \boldsymbol{s}} e^{-E_i \tau} \, \mathcal{A}^*{}_i \mathcal{A}_i \, \mathcal{M}_i^{\boldsymbol{s}} \, ,$$

$$\sum_{\boldsymbol{s}} \mathcal{M}_i^{\boldsymbol{s}} = e^{i\alpha_i \gamma_5} \frac{(-i \not{p}_i + M_i)}{2E_i^p} e^{i\alpha_i^* \gamma_5} = e^{i\alpha_i \gamma_5} \sum_{\boldsymbol{s}} u_N^i(\boldsymbol{p}, \boldsymbol{s}) \overline{u}_N^i(\boldsymbol{p}, \boldsymbol{s}) e^{i\alpha_i^* \gamma_5}$$

With CPV

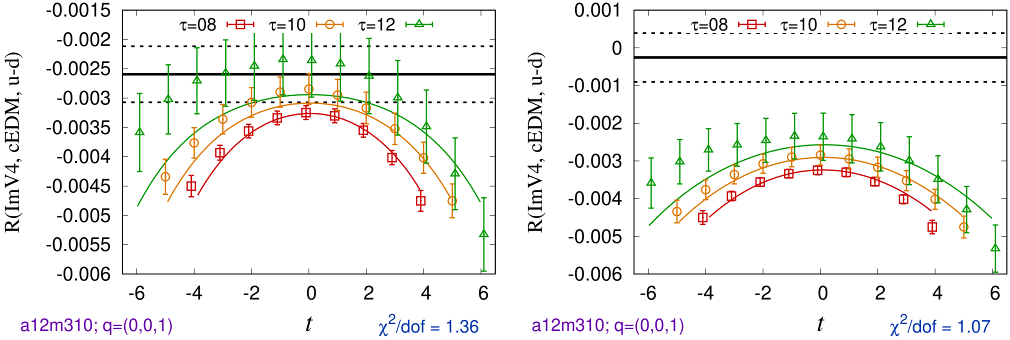
- γ_4 is no longer the parity operator for the neutron state
- There is a unique α for each
  - Nucleon interpolating operator N,
  - State created by N
  - CPV interaction

Excited-state Artifacts

All states with nucleon quantum numbers are created by N

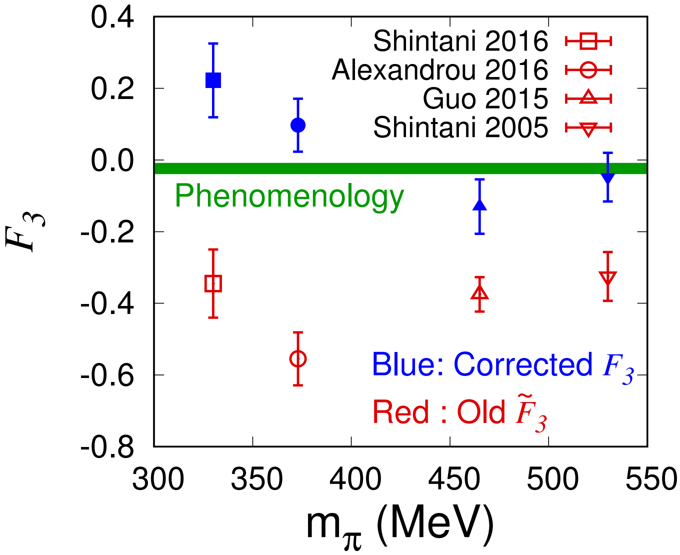
The excited state spectrum cannot be determined from 3-pt functions.

Systematic errors from possible enhancement of light $N\pi$ states in the 3-pt functions

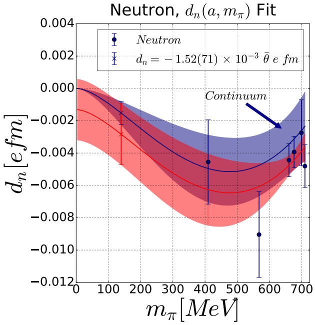


Calculations of the Θ-term pre 2017

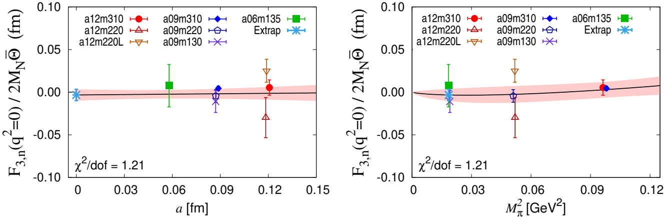
Abramczyk, *et al.* clarified the issue of $\alpha \Longrightarrow$ previous lattice give $d_n \approx 0$



Recent calculations with the Θ-term



Dragos, *et al.* (2019)



Bhattacharya, *et al.*, (2020)

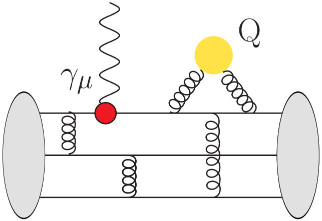
[Dragos, *et al.* (2019)]:

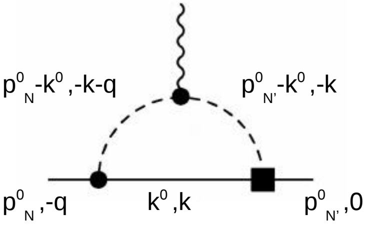
- multiple a but large pion mass $m_\pi > 400\mathrm{MeV}$
- $d_n = -1.52(71) \times 10^{-3}\ \bar{\theta}\ e \cdot fm$
- Inflection point occurs near smallest M_π to satisfy $d_n = 0$ at $M_\pi = 0$

Does the $N\pi$ excited state contribute?

Bhattacharya *et al.* (2021) perform a χPT analysis:

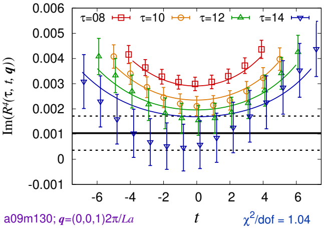
$\Longrightarrow$ Contribution of low energy $N\pi$ excited-state should grow as $M_\pi \to 135$ MeV



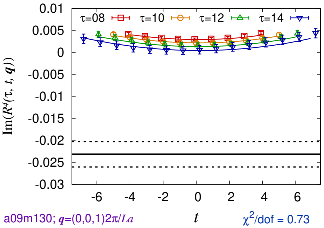


Including the $N\pi$ state gives a very different value for ground-state matrix element

$M_2 \sim 2\,\mathrm{GeV}$



$M_2 \sim 1\,\mathrm{GeV} + M_\pi$



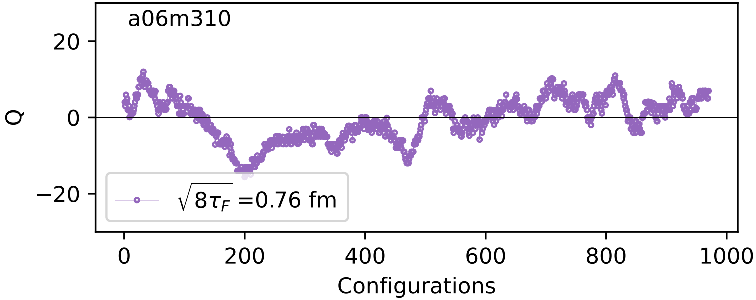
Status from Bhattacharya *et al.* (2021)

	Neutron $\overline{\Theta}$ e · fm	Proton $\overline{\Theta}$ e · fm
Bhattacharya 2021	$d_n = -0.003(7)(20)$	$d_p = 0.024(10)(30)$
Bhattacharya 2021 with $N\pi$	$d_n = -0.028(18)(54)$	$d_p = 0.068(25)(120)$
ETMC 2020	$\|d_n\| = 0.0009(24)$	–
Dragos 2019	$d_n = -0.00152(71)$	$d_p = 0.0011(10)$
Syritsyn 2019	$d_n \approx 0.001$	–

Table: Summary of lattice results for the contribution of the Θ-term to the neutron and proton electric dipole moment.

- No reliable estimate of the contribution of the Θ-term to nEDM
- Including the contribution of the lowest energy $N\pi$ excited state gives a much larger result

QCD θ-term future: All lattice systematics need better control



- Simulate on small a lattices to reduce discretization artifacts
- Simulate near $M_\pi = 135$ MeV
- Check for long autocorrelations in Q. These increase as $a \to 0$
- High statistics needed
- Resolve the contribution of $N\pi$ excited state
- Chiral-continuum fits

New algorithms needed for lattice generation at $a \lesssim 0.6$ fm to get high statistics

Neutron EDM from quark chromo EDM (qcEDM)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} &= -\frac{g_s^2}{32\pi^2}\overline{\theta} G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
&- \frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
&- \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
&+ d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
&+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

Lattice QCD approaches for qcEDM

$$S = S_{QCD} + S_{qcEDM}; \qquad S_{qcEDM} = \frac{g_s}{2} \sum_{q=u,d,s} \tilde{d}_q \int d^4x \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three different approaches developed
  - Schwinger source method [Bhattacharya, *et al.* (2016)]:

$$D_{clov} \to D_{clov} + \frac{i}{2}\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

  - Direct 4-point method with expansion in $\sum_q O_{qcEDM}$ [Abramczyk, *et al.* (2017)]:

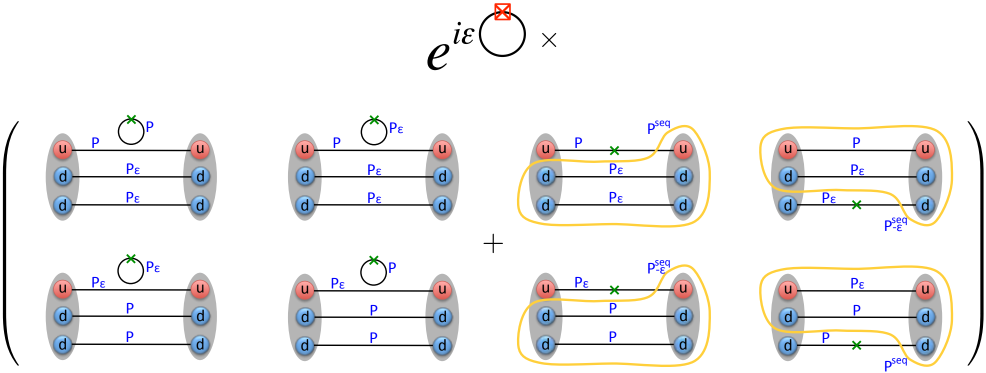
$$\langle N V_\mu \overline{N} \rangle_{qcEDM} = \langle N V_\mu \overline{N} \rangle + \tilde{d}_q \langle N V_\mu \overline{N} \sum_q O_{qcEDM} \rangle + \mathcal{O}(\tilde{d}_q^2)$$

  - External electric field method [Abramczyk, *et al.* (2017)]:

$$\langle N \overline{N} \rangle_{qcEDM}(\vec{\mathcal{E}}, t) = \langle N(t) \overline{N}(0) O_{qcEDM} \rangle_{\vec{\mathcal{E}}}$$

Three-point functions in the Schwinger Source Method

Quark propagators calculated with (P_ϵ) and without (P) the qcEDM operator with coupling ϵ added to the QCD action. These are contracted to form the following quark-line diagrams.



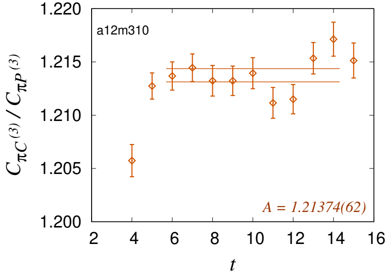
ϵ has to be small to avoid multiple insertions of qcEDM from P_ϵ.

Quark chromo-EDM operator has power-divergent mixing

$$\tilde{C} = i\bar{\psi}\Sigma^{\mu\nu}\gamma_5 G_{\mu\nu}T^a\psi - i\frac{A}{a^2}\bar{\psi}\gamma_5 T^a\psi$$

Demanding $\langle\Omega|\,\tilde{C}\,|\pi\rangle = 0$ fixes A:

$$\alpha_N(\tilde{C}) = 0 \implies \frac{1}{A}\frac{\alpha_N(C)}{\alpha_N(\gamma_5})) = 1$$



Ensemble	c_{SW}	a (fm)	t-range	A
a12m310	1.05094	0.1207(11)	6–14	1.21374(62)
a12m220L	1.05091	0.1189(09)	7–14	1.21800(33)
a09m310	1.04243	0.0888(08)	8–22	0.99621(30)
a06m310	1.03493	0.0582(04)	14–30	0.77917(24)

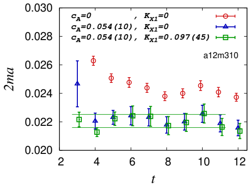
Multiplicative renormalization of qcEDM operator

Isovector pseudoscalar can be rotated away up to $O(a)$ effects!
We can determine the $O(a)$ effects non-perturbatively:

$$\frac{\langle \pi \left[a\partial_\mu A^\mu - \bar{c}_A a^2 \partial^P + \bar{K}(a^2 C - AP) \right] \rangle}{\langle \pi P \rangle} = 2\bar{m}a(1 + O(a^2))$$

So, on-shell zero-momentum

$$\text{M.E. of } P = \text{M.E. of } \frac{x \equiv a^2 \bar{K}}{y \equiv 2\bar{m}a + A\bar{K}} C \,.$$



Ensemble	fit-range c_A	fit-range $\bar{K}_{X1}$	$\chi^2/\mathrm{d.o.f}$ c_A	$\chi^2/\mathrm{d.o.f}$ $\bar{K}_{X1}$	c_A	$\bar{K}_{X1}$	$2\bar{m}a$	$\frac{2\bar{m}a}{K_{X1}}$	$\frac{2\bar{m}a}{2ma + AK_{X1}}$
a12m310	4–11	3–11	0.66	0.88	$0.054(10)$	$0.097(45)$	$0.02205(46)$	$0.23(10)$	$0.158(58)$
a12m220L	4–11	3–11	2.08	3.09	$0.0342(77)$	$0.183(35)$	$0.01152(21)$	$0.063(12)$	$0.0491(86)$
a09m310	5–15	4–15	0.99	1.09	$0.0277(40)$	$0.047(15)$	$0.01684(15)$	$0.35(11)$	$0.263(61)$
a06m310	6–20	5–20	0.29	1.53	$0.0093(17)$	$0.0272(60)$	$0.010460(37)$	$0.385(87)$	$0.331(50)$

qcEDM: Future Prospects

- Working on renormalization and operator mixing using the gradient flow scheme
- Signal in F_3
- Need algorithm developments for large scale simulations at physical pion mass and lattice spacing $a < 0.09$ fm
- Investigating machine learning methods to reduce computational cost

  [Yoon, Bhattacharya, and Gupta (2019)]

Renormalization using Gradient Flow

Gradient flow [Lüscher and Weisz (2011)]:

$$\partial_t B_\mu(t) = D_\nu G_{\nu\mu}, \qquad B_\mu(x, t=0) = A_\mu(x),$$
$$\partial_t \chi(t) = \Delta^2 \chi, \qquad \chi(x, t=0) = \psi(x)$$

- Smear (flow) gluon and quark fields along the gradient of an action to a fixed physical size (sets ultraviolet cutoff of the theory)
- The flowed operators have finite matrix elements except for an universal Z_ψ $\longrightarrow$ Allow us to take continuum limit without power-divergent subtractions
- Mixing and connection to $\overline{\mathrm{MS}}$: simpler perturbative calculation in continuum
- Calculations for CPV ops underway [Rizik, Monahan, and Shindler (2020)]

Neutron EDM from Weinberg's ggg and Various Four-quark Ops

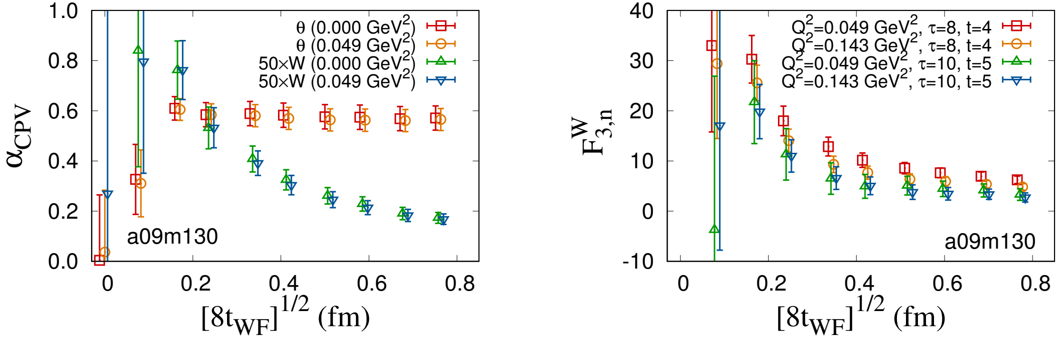
$$
\begin{aligned}
\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\bar{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

Weinberg's $G\widetilde{G}$ Operator: Status and Future Prospects

$$\mathcal{L}_{W_{ggg}} = \frac{1}{6} d_w g_s G\tilde{G}G$$

- Numerical Calculation is almost the same as for the QCD θ-term
- No publications yet, only a few preliminary studies
  [Yoon, Bhattacharya, Cirigliano, and Gupta (2019)]
- Signal is noisier than QCD θ-term
- Suffers from the long autocorrelations on $a \lesssim 0.06$ fm lattices
- Requires solving operator renormalization and mixing with the Θ-term
  - RI-MOM scheme and its perturbative conversion to $\overline{\mathrm{MS}}$ is available
    [Cirigliano, Mereghetti, and Stoffer (2020)]
  - Gradient flow scheme is being investigated to address divergent mixing structure
    [Rizik, Monahan, and Shindler (2020)]

Weinberg's $G\widetilde{G}G$ Operator: Mixing with the Θ-term



$1/t_{\rm WF}$ mixing with the Θ-term

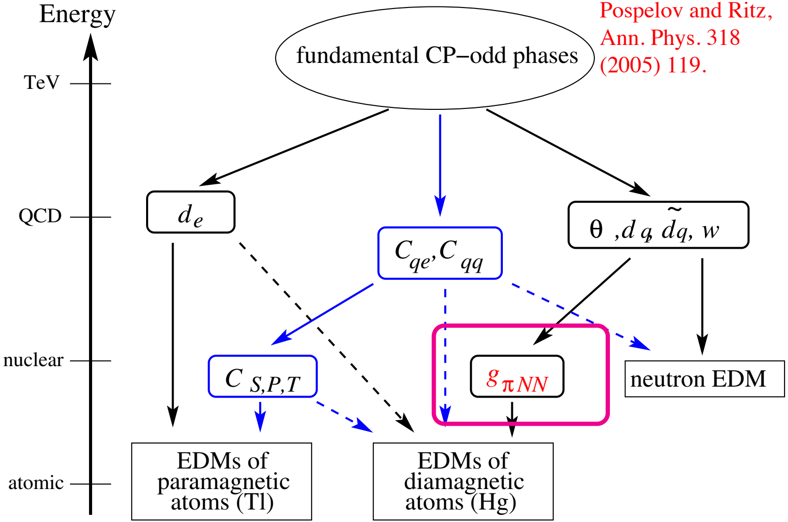
Four-quark operators: Current Status and Future Prospects

$$\mathcal{L}_{4q} = \sum_i C_{ij}^{(4q)} (\bar{\psi}_i \psi_i)(\bar{\psi}_j i\gamma_5 \psi_j) + \cdots$$

- No lattice QCD calculations yet!
- Calculation expected to be statistically noisy and computationally expensive
- Hopefully we can include this calculation in a long range (5–10 year) plan

Lattice Calculations for $g_{\pi NN}$

Pospelov and Ritz, Ann. Phys. 318 (2005) 119.



$g_{\pi NN}$: Current Status and Future Prospects

$$\mathcal{L}^{CPV}_{\pi NN} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N - \frac{\bar{g}_1}{2F_\pi}\pi_0\bar{N}N - \frac{\bar{g}_2}{2F_\pi}\pi_0\bar{N}\tau^3 N + \cdots$$

- Chiral symmetry relations + nucleon σ-term & mass splittings $\longrightarrow g_{\pi NN}$
  [Vries, Mereghetti, Seng, and Walker-Loud (2017)]
- No direct lattice calculation of $g_{\pi NN}$ published yet

Can be calculated from $\langle N|A_\mu(q)|N\rangle_{\mathrm{CPV}}$ following the same methodology used for neutron EDM via $\langle N|V_\mu(q)|N\rangle_{\mathrm{CPV}}$

Conclusion

- Significant progress, issues of signal, statistics and renormalization remain
- Gradient flow scheme is, so far, best for renormalization
- quark-EDM: Lattice QCD provides results with $\lesssim 5\%$ uncertainty
- Θ-term: Significant Progress. No reliable estimates yet
  - 10X Statistics
  - Does $N\pi$ provide leading excited-state contamination?
- quark chromo-EDM: Signal-to-noisemethods
  - Renormalization and mixing (Understood this for the isovector case)
  - Does $N\pi$ provide leading excited-state contamination?
- Weinberg $G\widetilde{G}G$ Operator: Signal
  - Address the mixing with Θ-term in gradient flow scheme
- Four-quark operators: Yet to be initiated

Could use 10x Larger Computational Resources